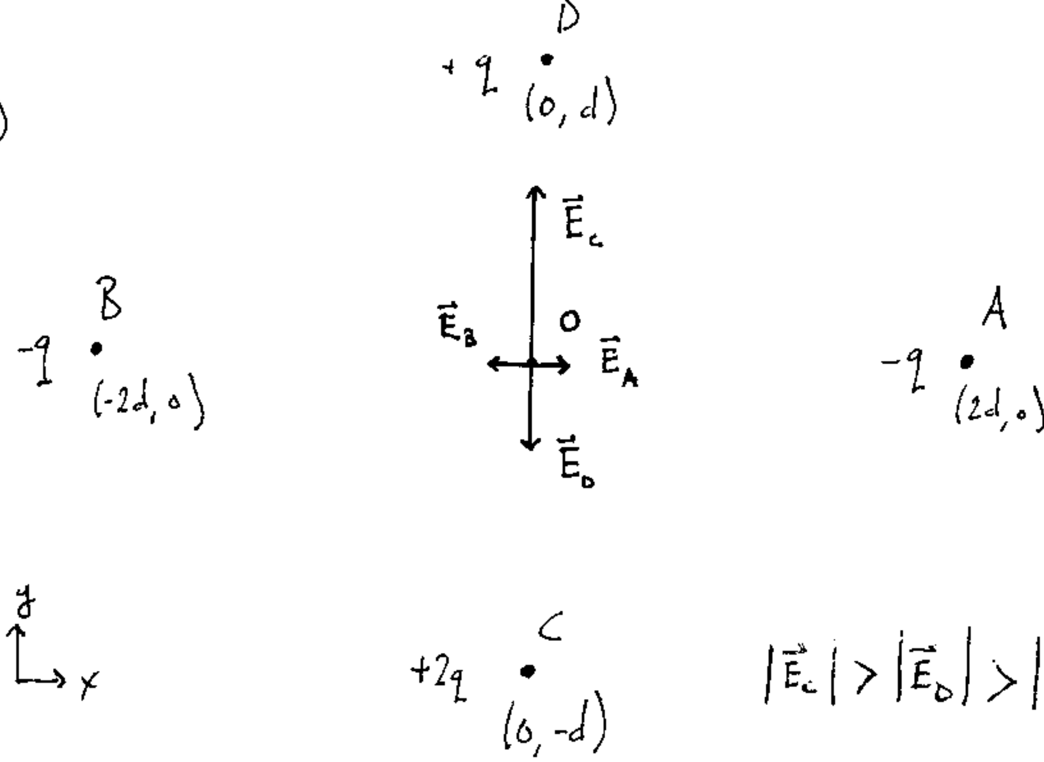
1)

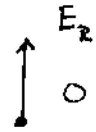
a)



$|\vec{E}_c| > |\vec{E}_D| > |\vec{E}_B| = |\vec{E}_A|$

Resultant electric field vector:

$$\vec{E}_R = \vec{E}_A + \vec{E}_B + \vec{E}_c + \vec{E}_D$$



b) Let $\vec{r}_{AO}$ be the displacement from A to O.

Since $|\vec{r}_{AO}| = |\vec{r}_{BO}|$ and $q_A = q_B$, we have that the electric field contributions from A and B are equal and opposite and thus cancel.

It remains to calculate the contributions from C and D:

$$\vec{E}_c = \frac{k(2q)}{d^2}\hat{r}_{co} = \frac{2kq}{d^2}\hat{y} \qquad \text{where } k = \frac{1}{4\pi\varepsilon_0}$$

$$\vec{E}_D = \frac{k(q)}{d^2}\hat{r}_{Do} = \frac{kq}{d^2}(-\hat{y})$$

$$\Rightarrow \vec{E}_{TOT} = \frac{2kq}{d^2}\hat{y} - \frac{kq}{d^2}\hat{y} = \boxed{\frac{kq}{d^2}\hat{y}}$$

c) In principle, one could obtain the electric field from the electric potential $V(\vec{r})$. Recall that the electric field is just the slope of the electric potential function. In one dimension,

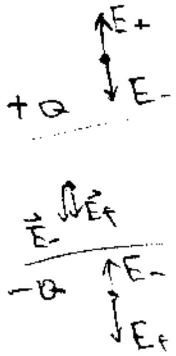
$$E(x) = -\frac{dV(x)}{dx}$$

Using the superposition principle, $V(\vec{r})$ can be calculated as $V(\vec{r}) = V_A(\vec{r}) + V_B(\vec{r}) + V_c(\vec{r}) + V_D(\vec{r})$

where $V_A(\vec{r}) = \frac{k\, q_A}{|\vec{r}_A|}$

is the potential from a point charge.

2: a) There are two plates, each of which generates an electric field which can be calculated with Gauss' Law. The total $\vec{E}$-field in each region follows from superposition priciple.



$$\vec{E}_{in} = \vec{E}_+ + \vec{E}_- = \frac{\sigma}{2\epsilon_0}(-\hat{j}) + \frac{\sigma}{2\epsilon_0}(-\hat{j}) = \frac{\sigma}{\epsilon_0}(-\hat{j})$$

$$= \frac{Q}{A\epsilon_0}(-\hat{j});$$

$$\vec{E}_{out} = \vec{E}_+ + \vec{E}_- = 0.$$

b) $\Delta V = -\int \vec{E} \cdot d\vec{r}$. For uniform field, the magnitude is $\Delta V = E \cdot d = \frac{Qd}{A\epsilon_0}$.

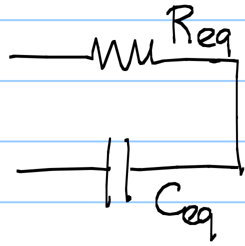
c) The definition of capacitance gives

$$C = \frac{Q}{\Delta V} = \frac{A\epsilon_0}{d}.$$

8b Fall '09 Bordel Midterm 1 Problem 3 Solution

Note Title 10/8/2009

(a) Equivalent circuit:



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} + R_3 = \frac{R(3R)}{R+3R} + \frac{R}{2} \Rightarrow \boxed{R_{eq} = \frac{5}{4} R}$$

$$C_{eq} = C_1 + C_2 \Rightarrow \boxed{C_{eq} = 3C}$$

(b) $V_{R_{eq}} = \mathcal{E} - V_{C_{eq}}$ (by Kirchoff's voltage-loop law)

$$\Rightarrow V_{R_{eq}}^{max} = \mathcal{E} \Rightarrow \boxed{P_{R_{eq}}^{max} = \frac{(V_{R_{eq}}^{max})^2}{R_{eq}} = \frac{4\mathcal{E}^2}{5R}}$$

(c) $V_{C_{eq}}^{max} = \mathcal{E} \Rightarrow \boxed{U_{C_{eq}}^{max} = \frac{1}{2} C_{eq} (V_{C_{eq}}^{max})^2 = \frac{3}{2} C\mathcal{E}^2}$

(d)(i) $I = C\frac{dV}{dt} \Rightarrow V_{C_{eq}}$ must be continuous $\Rightarrow$ since $V_{C_{eq}} = 0$ right before the battery is connected, then it is 0 right after $\Rightarrow$ all the voltage $\mathcal{E}$ is initially dropped across R_{eq} $\Rightarrow$ $\boxed{I = \frac{\mathcal{E}}{R_{eq}} = \frac{4\mathcal{E}}{5R} \text{ (short time)}}$

<u>Note</u>: this is the current through the resistor, but since it is in series with the capacitor, this is also the current through the capacitor.

Since $V_{C_{eq}} = 0$ initially, $\boxed{Q_{C_{eq}} = 0 \text{ (short time)}}$

(ii) As $t \to \infty$, the circuit settles down so that $\frac{dV}{dt} = 0$

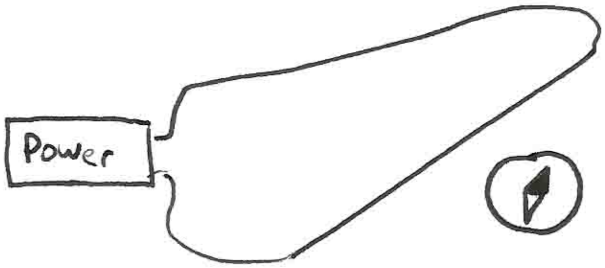
$\Rightarrow \boxed{I = 0 \text{ (long time)}} \Rightarrow V_{R_{eq}} = IR_{eq} = 0 \Rightarrow V_{C_{eq}} = \mathcal{E}$

$\Rightarrow \boxed{Q_{C_{eq}} = C_{eq}\mathcal{E} = 3C\mathcal{E} \text{ (long time)}}$

Problem 4 (10 Points)

a) Oersted's experiment used a compass to demonstrate the existence of a magnetic field created by a current.

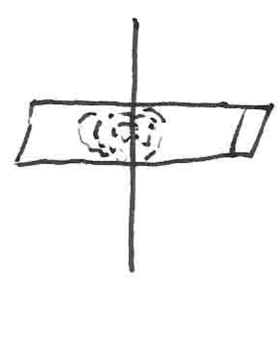
To recreate this experiment, you would want a compass, some wire, and some current source: a battery, power supply, etc.



The basic idea is to show that the compass (itself a small magnet) is affected by the presence of a current-carrying wire. Thus, you could turn the power supply off, move the compass around the wire, and see no effect. Then, after turning the power supply on, the compass will be suddenly affected by the presence of the wire, indicating that a magnetic field is created. Other possible answers include using a magnet in place of the compass, iron filings, using a solenoid...

b) To visualize the field lines, you could:

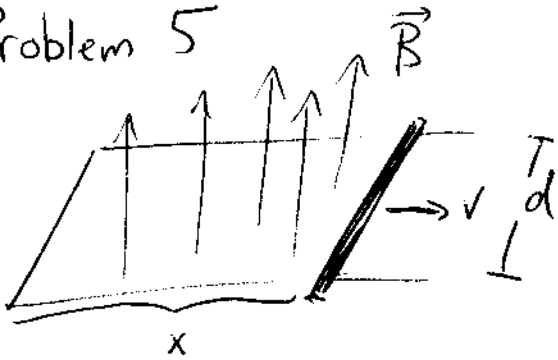
- use a compass to trace out the direction of $\vec{B}$ as you move it around the wire
- use iron filings to visualize the field:
  - viewing paper



There are other correct answers as well.

(Right hand rule, etc)

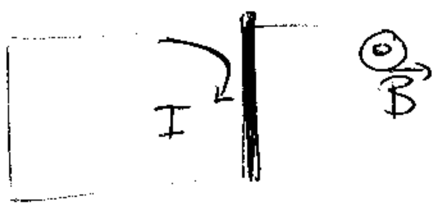
Problem 5



(a) $\Phi_B = \vec{B}\cdot\vec{A} = BA = Bdx$, $x(t) = vt$, $\Phi_B(t) = Bdvt$

(b) $\mathcal{E}_{emf} = -\frac{d\Phi_B}{dt} = -Bdv$. By Lenz's law, the induced current generates a field anti-parallel to $\vec{B}$. By RHR, current flows clockwise as viewed from above. It has magnitude

$$\frac{Bdv}{R}$$



(c) $\vec{F} = I\vec{L}\times\vec{B}$. $|\vec{F}| = \frac{Bdv}{R}\cdot dB = \frac{(Bd)^2 v}{R}$

By RHR, $\vec{F}$ directed opposite to $\vec{v}$. This agrees with Lenz's law since the force opposes the motion and thereby resists the increase in flux lines through the frame.

(d) $W = |\vec{F}|L = \frac{(Bd)^2 v}{R} L$. Alternatively,

$$P = I^2R = \left(\frac{Bdv}{R}\right)^2 R = \frac{(Bd)^2 v^2}{R} = \frac{\Delta E}{\Delta t}, \quad \Delta E = \frac{(Bd)^2 v (v\Delta t)}{R}$$

$$= \frac{(Bd)^2}{R} vL$$