1 a. $k = \frac{2\pi}{\lambda} = \frac{2\pi}{\left(60\text{cm} \cdot \frac{1\text{m}}{100\text{cm}}\right)} = \frac{2\pi}{0.6\text{m}} = 10.5\ \text{m}^{-1}$

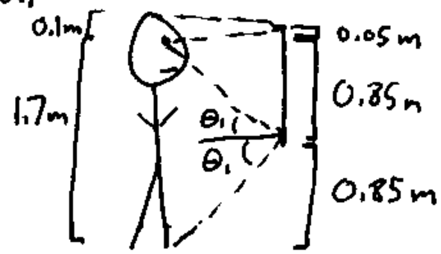
b. $f = c/\lambda = 3 \cdot 10^8 \frac{\text{m}}{\text{s}} / 0.6\text{m} = 5 \cdot 10^8\ \text{Hz}$

c. $\omega = 2\pi f = 3.14 \cdot 10^9\ \frac{\text{rad}}{\text{s}}$

d. $T = 1/f = 2\ \text{ns}$

Problem 2

a)



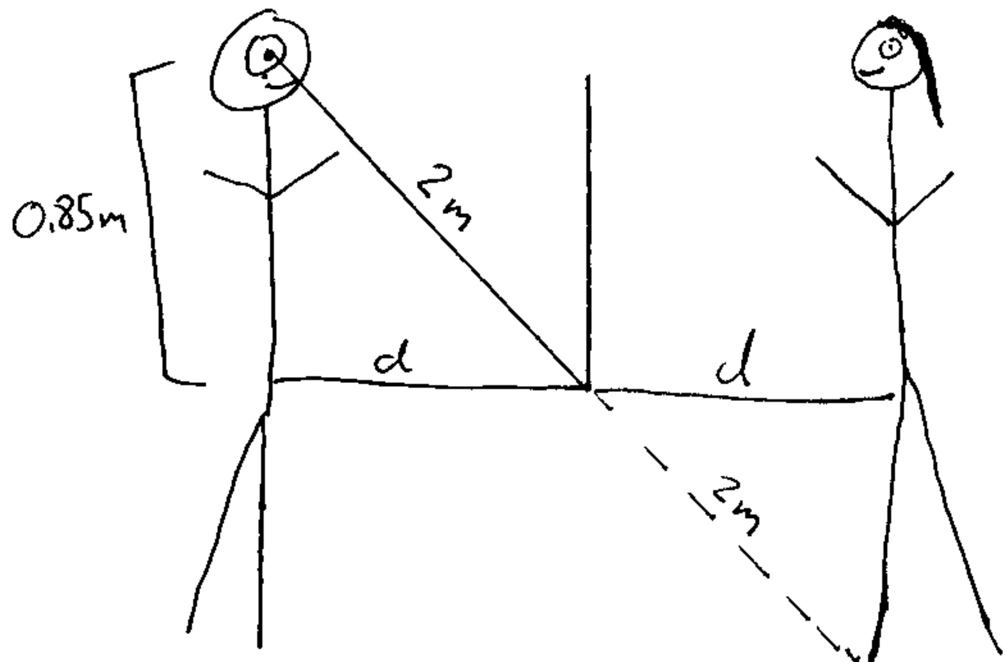
The law of reflection requires that the woman's line of sight hits the mirror halfway (vertically) between her eyes (at 1.7m) and what she is to see (her head at 1.8m, or her feet at 0m)

Thus, $h_{\text{bottom of mirror}} = 1.7\text{m} - \frac{1.7\text{m} - 0\text{m}}{2} = 0.85\text{ m}$

$h_{\text{top of mirror}} = 1.7\text{m} - \frac{1.7\text{m} - 1.8\text{m}}{2} = 1.75\text{ m}$

length of mirror $= 1.75\text{ m} - 0.85\text{m} = 0.9\text{ m}$

b) Remember that, for a plane mirror, the image is virtual and the same distance from the mirror as the object. Additionally, the woman wants to be able to see how her nice shoes look, not just her eyeglasses, so we will need the following diagram

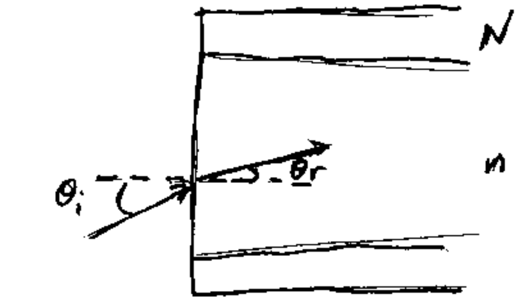


(Apologies to this poor woman whose proportions are greatly distorted)

The Pythagorean formula says

$$0.85^2 + d^2 = 2^2 \rightarrow d = \sqrt{4 - 0.85^2} = 1.81\text{ m}$$

Problem 3

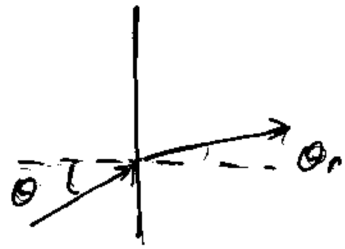


a) 5pts

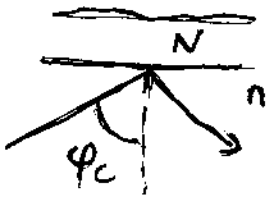
$$n_{out} \sin(\theta_i) = n \sin\theta_r$$

$$\Downarrow$$

$$\boxed{\sin\theta_i = n \sin\theta_r}$$



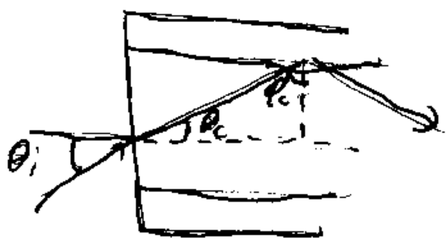
b) 5pts



$$\boxed{\sin\varphi_c = \frac{N}{n}}$$

for $N = 1.4$, $n = 1.5$ this gives $\varphi_c = 69° = 1.2$ radians

c) 10pts



$$\theta_r + \varphi_c = 90° = \pi/2$$

$$\sin\theta_i = n \sin\theta_r$$

$$= n\left(\sin\left(\pi/2 - \varphi_c\right)\right) = n\cos(\varphi_c)$$

$$\boxed{= n \sin\left(\pi/2 - \sin^{-1}(N/n)\right)} = n\cos\left(\sin^{-1}(N/n)\right)$$

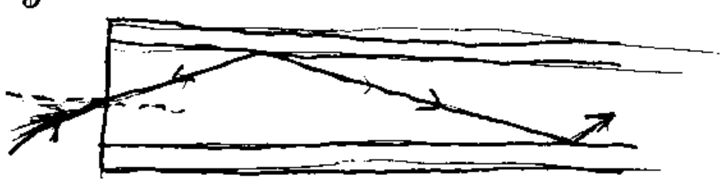
$$\Rightarrow n \cdot \frac{\sqrt{n^2 - N^2}}{n}$$

$$= \sqrt{n^2 - N^2}$$

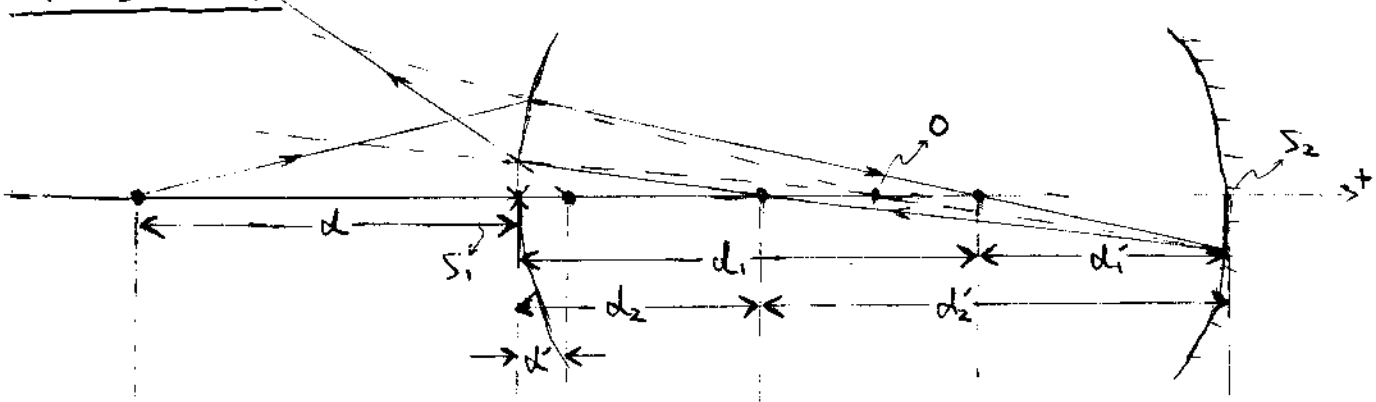
$$\boxed{\theta = \sin^{-1}\left(\sqrt{n^2 - N^2}\right)}$$

For $N = 1.4$, $n = 1.5$, this gives $\boxed{\theta_i = 32.58°}$

for $\theta_i < 32.58°$, we get total internal reflection



Problem 4



(a) When the ray enters the left surface: refraction; (R1)
When the ray meets the right surface: reflection; (L)
When the ray exits the left surface: refraction. (R2)

(b) In the small-angle approximation,

for reflection: $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$ (31.2)
for refraction: $\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$ (31.6)
TEXTBOOK

As applied to this problem, with the distances as labelled

$$\text{(R1)}: \frac{1}{d} + \frac{n}{d_1} = \frac{n-1}{R} \quad ①$$

$$\text{(L)}: \frac{1}{d_1'} + \frac{1}{d_2'} = \frac{2}{R} \quad ②$$

$$\text{(R2)}: \frac{n}{d_2} + \frac{1}{d'} = \frac{1-n}{-R} \quad ③$$

In addition, there are two geometric relations

$$d_1 + d_1' = 2R, \quad ④$$

$$d_2 + d_2' = 2R. \quad ⑤$$

(c) The object's position $d_o = d$ and the image's position $d_i = d'$.
We have 5 equations for 5 unknowns — d_1, d_1', d_2', d_2, d'.
After some lengthy algebra, we find in the end

$$d' = \frac{R(4R + d(4-n))}{n(R+2d) - 4(R+d)}$$

(d) $d = 20$ cm; $R = 10$ cm; $n = 1.5$. $\Rightarrow$ $d' = -20$ cm

Image A' forms at 20 cm to the <u>right</u> of S_1 (virtual).

Problem 5

(a) (i) The normal eye has a near point of 25 cm. We want images to appear 1.5 cm behind the lens (on the retina) and so $\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f}$ implies $\frac{1}{S_o} = \frac{1}{f} - \frac{1}{1.5\text{cm}} = \frac{1.5\text{cm} - f}{f \cdot 1.5\text{cm}}$

The object distance must be positive, so $1.5\text{ cm} \geq f$

Furthermore, $25\text{cm} \leq S_o \Rightarrow \frac{1}{25\text{cm}} \geq \frac{1}{f} - \frac{1}{1.5\text{cm}}$ or

$\frac{1}{25\text{cm}} + \frac{1}{1.5\text{cm}} \geq \frac{1}{f}$. Hence $\left(\frac{1}{25\text{cm}} + \frac{1}{1.5\text{cm}}\right)^{-1} \leq f \leq 1.5\text{ cm}$

(ii)

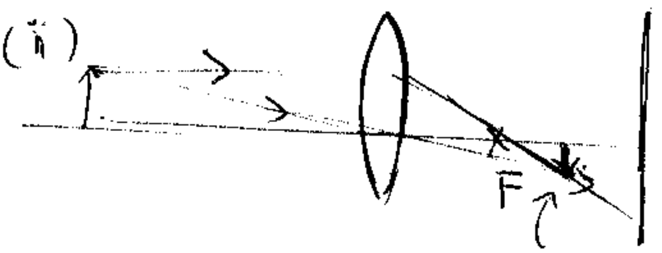


image (near focal point)

(iii) The near point is 25 cm $\Rightarrow S_o \geq 25\text{cm}$

(b)(i) Myopia or near sightedness

(ii) A divergent lens, i.e. convex meniscus, is needed to correct myopia

Want objects at infinity to be imaged 80 cm in front of meniscus:

$\frac{1}{f_c} = -(80\text{cm})^{-1} \Rightarrow f_c = -80\text{cm}$. $P = \frac{1}{f} = -\frac{1}{80\text{cm}} = -\frac{1}{.8}$ diopters

(iii) Let S_i' be the image of an object viewed through corrective lens

Want $\frac{1}{|S_i'|} + \frac{1}{1.5\text{cm}} = \frac{1}{f} = -\frac{1}{S_i'} + \frac{1}{1.5\text{cm}} \Rightarrow \frac{1}{S_i'} = \frac{1}{1.5\text{cm}} - \frac{1}{f} = \frac{1}{f_c} - \frac{1}{S_o}$

$\Rightarrow \frac{1}{S_o} = \frac{1}{f_c} + \frac{1}{f} - \frac{1}{1.5\text{cm}} \leq \frac{1}{f_c} + \frac{1}{25\text{cm}} = \frac{1}{-80\text{cm}} + \frac{1}{25\text{cm}} = \frac{(25-80)\text{cm}}{-80 \cdot 25(\text{cm})^2}$

or $S_o \geq 36.36$ cm ← near point