Carbon Dating
The decay of radioactive nuclei can be used to measure the age of artifacts, fossils, and rocks. The half-life of C^{14} is 5730 years.

a) If a sample shows only one-fourth of its estimated original quantity of C^{14}, how old is it? Think about half-lives; don't try to just plug into the decay equation.

b) Suppose that an Egyptian farmer claims to have discovered a linen burial cloth used during Egypt's Middle Kingdom some 4000 years ago. Careful analysis shows that the cloth contains 80% of the C^{14} that it is estimated to have originally contained. Is the cloth 4000 years old?

c) How does one estimate the amount of C^{14} that would have been originally contained in an object? Why does that work?

Radiation Doses
Radioactive isotopes are often used for medical imaging. Technetium-99m (99Tc) is a particularly popular isotope since it has a short half-life and is easy to make. It has a half-life of 6 hours. It decays by emitting a gamma ray, with energy 140keV. For this problem, assume that it is present in the body for 12 hours after being injected and then is immediately removed. A typical amount of 99Tc injected for a medical scan is 5×10^8 decays per second, also known as becquerels (Bq), the SI unit of activity.

a) What is the number N_0 of 99Tc atoms needed to have this much activity?

b) How many atoms decay during the 12 hours the isotope is present in the body?

c) The gamma ray released by each decay carries 140keV of energy. Find the total energy E_0 released by decays in the 12 hours. (You don't have to multiply out all the factors, but make sure they're all present; the answer must be in SI units)

d) If the amount of 99Tc that you calculated in Part C were administered to an 80-kg man, staying in his body for 12 hours as described, what would his absorbed dose be in rads?

e) Equivalent dose correlates strongly with the biological effects of radiation. Calculate the equivalent dose, keeping in mind that 99Tc emits gamma rays. How would the equivalent dose change if 99Tc emitted an alpha particle of the same energy?

Optics (20 pts)

A small object is placed near a concave spherical mirror. The object is 3/4 of the focal length in front of the mirror.

a) Using ray tracing, determine where the image will be located. Note that you must convince us that you have located an image, not just a place where two lines cross.

b) Calculate the exact position of the image. Is this consistent with your ray diagram?

c) Calculate the height of the image in terms of the height of the object (e.g. "12.7 times as high as the object"; that's not the right answer!) Is this consistent with your ray diagram?

Complex Circuits

a) A resistor R, an inductor L, and a capacitor C are connected in parallel to a sinusoidal source of voltage $V(t) = V_0 \sin \omega t$. Sketch the voltage and current in each of the three components so we can tell their phase and amplitude.

b) A resistor R, an inductor L, and a capacitor C are connected in series to a sinusoidal source of current $I(t) = I_0 \sin \omega t$. Sketch the voltage and current in each of the three components so we can tell their phase and amplitude.

c) What is the magnitude of the total voltage across the three elements in part b as a function of R, L, C and ω?

d) For some value of ω, this total voltage will have a minimum magnitude. What is this minimum magnitude?

Writing on the Moon
You're tasked with using a laser to write an advertising logo on the moon. To succeed, you're told you need to make a spot on the moon (400×10^3 km away) that's about as bright as daylight on the moon (10^3 w/m^2) and about a kilometer in diameter. You have access to a telescope with an aperture (main lens diameter) of 2m. You'll be using a laser with a wavelength of 500nm (green).
   a) How much power does your laser need to make the km-sized spot? Is this going to be easy or hard?
   b) All that power has to leave via the 2m diameter main lens of your telescope. What's the peak electric field in the light there?
Assuming that the telescope, laser, etc are as good as can be done within the laws of physics, what's the smallest spot you can make on the moon? What's the physical limitation that keeps you from making it smaller?

AC Dimmers
In class, we demonstrated a way of dimming AC lights using various circuit components. We'll examine that in detail here. You're using a 100V 60 Hz AC supply, a 50 W light bulb, and variable resistors, capacitors and inductors.

   a) When the bulb is connected to the power, no other elements involved, how much current does it draw? (We'll call this "full on") What is its resistance?

b) What resistor would you have to put in series with the bulb so that the bulb only produces 25W, e.g. "half bright"? Would a larger resistor result in a brighter or dimmer bulb?
c) How much power is dissipated in that added resistor?
d) Putting an inductor in series with the bulb can also dim it. What value of inductor is needed to make the bulb half bright? Would a larger inductor result in a brighter or dimmer bulb?
e) How much power is dissipated in that inductor?
f) Can you dim the bulb by putting a capacitor in series with it? If not, why not. If so, would a larger capacitor make the bulb brighter or dimmer?

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Fields in a Nerve Fiber
Assume that a nerve fiber is a long tube surrounding a conductive liquid. The tube is a non-conducting membrane (cell wall) with an inner diameter of R_i and an outer diameter of R_o. You are able to measure the voltage V_m between the inside and outside of the membrane. (V_m may be zero or a positive or negative number)

a) On the figure, show where positive and negative charges will be if V_m is less than zero at some instant.
b) Find the linear charge density (C/m) present as a function of V_m and the dimensions.
c) For a given positive V_m, find the electric field inside the liquid. Be sure to specify both magnitude and direction (if needed).
d) For a given positive V_m, find the electric field inside the membrane. Be sure to specify both magnitude and direction (if needed).
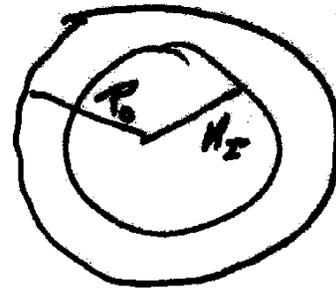
Physical Optics
During the interference lab, you looked at the interference patterns made by hairs and blood cells. For simplicity, assume that the laser you used had a wavelength of 600nm.

a) Describe how you would measure the width of a single hair using diffraction. (As part of this, you should draw and label a picture, including dimensions).
You also looked at slide containing a large number of small disks: blood cells. You shined a laser on this slide, illuminating lots of cells, and got a circular diffraction pattern. Describe how you would measure the typical size of a blood cell using diffraction (draw a picture, etc). Be sure to discuss the fact that there are lots of blood cells being illuminated and creating separate diffraction patterns. How does that effect the answer?

Bombarding the Cesium Atom (25 pts)

The figure shows the energy levels of the element cesium. Electrons of specific energies collide with gaseous cesium atoms. Consider only energy during the collisions; don't worry about momentum.
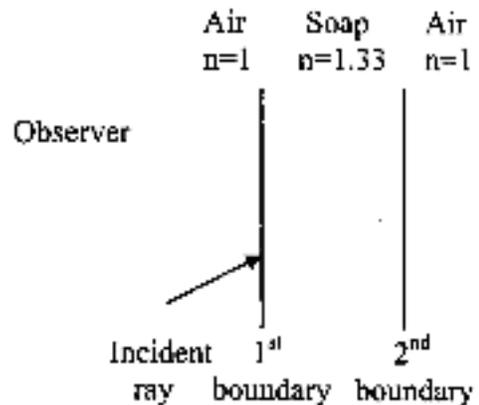
a) In what state are the atoms of a cold (room temperature or below) gas of cesium atoms before the collisions?

b) What are the possible energies of the electrons emerging if the energy of the colliding electrons is:
I) 1.0eV?
II) 2.0eV?
III) 2.6 eV?
IV) 3.2ev?

c) An electron of energy 1.8eV collides with a cesium atom in the ground state. After the collision the electron is found to have 0.4eV of energy. Relative to the ground state, what is the energy of the cesium atom after the collision?

d) What is the minimum energy that a bombarding electron must have in order to excite a cesium atom? What is the minimum energy that a bombarding electron must have in order to ionize a cesium atom?

Ionization

3.9eV ————————————————————————

2.4eV ————————————————————————

1.4eV ————————————————————————

0eV ————————————————————————
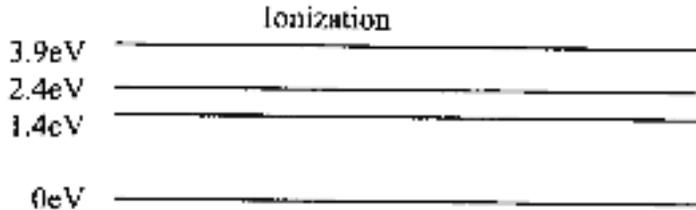
Soap Film (25 pts)

Light of wavelength λ is incident from the left on a soap film on thickness d. (See figure)

a) What is the frequency, velocity and wavelength of the light in the soap film?

b) Find the smallest value of d for which the observer on the left will see all of the light reflected back. Be sure to explain this result, not just state it.

c) If the film has the thickness found in part a, how much light is passing through to the film to the right? Why?

d) Will the observer still see a bright reflection if the light is changed to a wavelength of 2λ? To a wavelength of $\lambda/2$?

Air   Soap   Air
n=1   n=1.33   n=1

Observer

Incident   1st 2nd
ray   boundary   boundary

Electron Beam (25 pts)

We want to form a beam of electrons by emitting them at effectively zero velocity from a source, accelerating them by allowing them to move through a voltage difference V, and then passing the moving electrons through a small hole of width D.

a) What is the energy of the electrons?
b) What is the momentum of the electrons?

Find the minimum angular spread that this beam could have after going through the small hole (hint: Consider using either the uncertainty principle or diffraction)

Laser surgery

A person's vision may be improved significantly by having the cornea reshaped with a laser beam, in a procedure known as photorefractive keratectomy. The excimer laser used in these treatments produces ultraviolet light with a wavelength of 193nm.

a) What is the difference in energy between the two levels that participate in stimulated emission in the excimer laser? Explain.
b) How many photons from this laser are required to deliver a total energy of 1.58 x 10^{-13}J to the cornea? Explain.