Fall 2009 8b Midterm 1 Review

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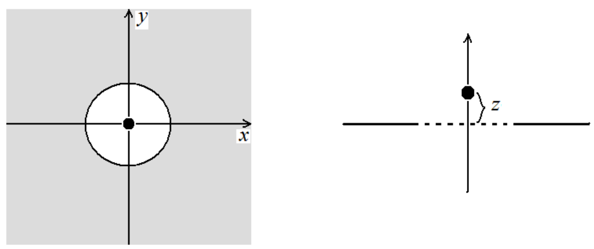
1 Charge Oscillator: Plane with a Hole



Top View Side View

In the x-y plane (at $z = 0$) lies an infinite plane of constant surface charge density $\sigma > 0$ with a hole of radius R centered at the origin cut out of it. At the center of this hole, we place a negative point charge $-q$ of mass m. We displace the point charge vertically a very small amount z ($z > 0$ means we pull the charge up and $z < 0$ means we push it down), and then we let go. Your task is to describe the subsequent motion of the charge. To do this, do the following

(a) First, suppose that there is no hole in the infinite plane. Recall or rederive using Gauss' Law the electric field produced at the position of the point charge at $(0, 0, z)$. Don't forget to write both the magnitude and direction.

(b) Now, calculate the electric field produced at the same point $(0, 0, z)$ by a uniformly charged disc of charge density $-\sigma$, radius R and placed in the x-y plane centered at the origin.

(c) Relate the electric field at the point $(0, 0, z)$ produced by the infinite plane with the hole to your answers from part (a) and (b).

(d) Now that you know the electric field at the position of the point charge, calculate the force felt by the point charge.

(e) Describe the motion of the charge after we let go.

2 Capacitors and Dielectrics

A parallel-plate capacitor has square plates of side length $L = 10$ cm and a separation of $d = 4$ mm. For the following, let the dielectric have constant $\kappa = 2$. Compute the capacitance in the following scenarios:

(a) No dielectric. Call this C_0.

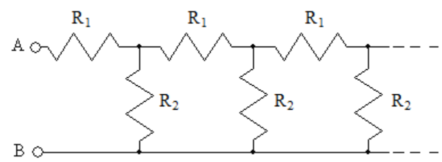
(b) Filled with dielectric. Call this C_1.

(c) Half-filled with a 10 cm $\times$ 10 cm $\times$ 2 mm slab of dielectric. Call this C_2.

(d) Half-filled with a 10 cm $\times$ 5 cm $\times$ 4 mm slab of dielectric. Call this C_3.

(e) Show that configuration (d) is at least "as good as" configuration (c).

3 Infinite Chain of Resistors



(*Purcell* 4.32, page 167) Some important kinds of networks are infinite in extent. The figure above shows a chain of resistors stretching off endlessly to the right. This is sometimes called an attenuator chain, or a ladder network. We would like to find the "input resistance," that is, the equivalent resistance between terminals A and B.

(a) Call the equivalent resistance of this network R. Add one more chain on the left (one more R_1 branching off to the left from point A and one more R_2 between points A and B). Calculate the equivalent resistance R' of this new circuit.

(b) Which of the following should be true: $R' > R$, $R' = R$ or $R' < R$? (Hint: have I changed the network by adding one more chain? If I have, how so?)

(c) Solve for R in terms of R_1 and R_2.

(d) Suppose I instead attach a battery of voltage V between terminals A and B. Call the voltage drop across the first (left-most) R_2 resistor V_1, then V_2 for the second R_2 and so on. Write V_n in terms of the voltage V of the battery, R_1, R_2 and R (the answer may or may not involve all of these).

(e) If I want the voltage V_n to be cut in half after each step along the chain (that is, $V_{n+1} = V_n/2$), then how must R_1 and R_2 be related and what is the equivalent resistance R of the chain in this case?

4 Moving Charges between Concentric Cylinders

Suppose I have two concentric conducting cylinders both of length L with the smaller one having radius a and the larger one having radius b. Suppose that $L >> b$ so that as far as the electric field is concerned, these are essentially infinitely long. These cylinders start out electrically neutral. How much work does it take to transfer charge from one to the other until they carry charges $\pm Q$? (Hint: review how you solved Problem 23.39 in problem set 3).

5 Expanding Loop in a Solenoid

Suppose we have a loop of conducting wire with resistance R having some initial area (note: the loop is not necessarily a circle; it could form any 2D (flat) shape). This loop is placed inside a solenoid of length L, which has N windings carrying current I_s. The loop always remains inside the solenoid and always lies flat inside the solenoid such that the plane of the loop is parallel with the planes of the windings of the solenoid. Suppose that I now somehow expand the loop of wire such that its area increases at the constant rate $\dot{A}$ (note: $\dot{A}$ is a constant having units m^2/s; also, the shape of the loop could very well be changing as long as the area is increasing at a constant rate; finally, I manage to do this without increasing the mass of the loop). I eventually stop after some time T has elapsed. How much work did I do? (Assume that the process of changing the area of the loop would not have cost any work had the loop not been in the magnetic field).

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6 Challenge Question: Magnetic Monopoles

Note: I will probably not present this problem during the review since it is also probably considerably more challenging than a typical exam question. But, it's here as a challenge question.

The field lines we have seen do not start or end anywhere unlike electric field lines that started and ended on point charges. However, we can still imagine a point-like object (like a point-charge, just without the electric charge) that is a source (or sink) of magnetic field lines. I believe that the scientific concensus is that we have never experimentally observed such an object, but we can still imagine what would happen if one did exist. Such an object is called a magnetic monopole (because it is like an object just having one magnetic pole rather than both North and South poles).

A magnetic monopole (MM) produces a magnetic field like the electric field of a point charge:

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0 g}{4\pi r^2}\,\hat{\mathbf{r}} \tag{6.1}$$

where g measures the magnetic "charge" of the MM (the magnetic analogue of electric charge) and where r is the radial distance from the position of the MM.

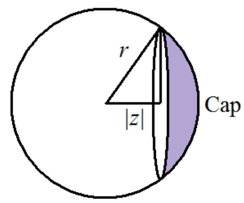
Imagine that we place a circular loop of wire of resistance R and radius R_ℓ in the x-y plane centered at the origin and that a MM with magnetic charge $g > 0$ travels along the z-axis from $z = -\infty$ (starting at time $t = -\infty$) to $z = +\infty$ (ending at time $t = +\infty$) and it passes through the loop at $t = 0$. The speed of the MM is v and is constant and much less than the speed of light (weird things would happen if it were moving too fast).

(a) Draw a picture of the situation when the MM is at position $z < 0$ and another one for $z > 0$. In both cases, draw the direction in which the MM is moving and draw the magnetic field lines of the MM making sure to depict which field lines contribute to the magnetic flux through the loop of wire.

(b) At any given moment, imagine freezing time when the MM is at position z (positive or negative). Imagine a sphere, S, centered at the MM and having radius equal to the distance between the MM and any point on the wire loop. Carefully draw this sphere S into one of your drawings from part (a). Compute the magnetic flux, Φ_S through the whole sphere S.

(c) The loop divides S into two pieces, S_1 and S_2 with S_1 having the generically smaller surface area of the two. We will call S_1 the "cap". Label these pieces in your diagram. Let A and A_1 be the surface area of S and S_1, respectively. Write A and A_1, which are functions of the position, z, of the MM along the z-axis, in terms of things given in the problem. Combine these with your answers from parts (b) and (c) to write $|\Phi_{S_1}|$ in terms of things given in the problem. (Note:

you will need the formula for the surface area of a "cap" of a sphere in terms of the radius of the sphere, r, and the distance, $|z|$, from the center of the sphere to the center of the circle that bounds that cap. This is $A_{\text{cap}} = 2\pi r^2(1 - \frac{|z|}{r})$.)



(d) Express $|\Phi_{S_1}|$, the magnitude of the magnetic flux through S_1, in terms of Φ_S, A and A_1. Use your answer to part (c) to write $|\Phi_{S_1}|$ in terms of things given in the problem.

(e) What is the relationship between $|\Phi_{\text{loop}}|$, the magnitude of the magnetic flux through the loop, and $|\Phi_{S_1}|$?

(f) Calculate the magnitude of the induced emf in the wire as a function of time, $|\mathcal{E}_{\text{ind}}(t)|$.

(g) Calculate the induced current in the wire as a function of time, $I(t)$. If you looked at the loop from above (i.e. such that the MM is moving directly towards you), in which direction would the induced current be flowing in the loop?