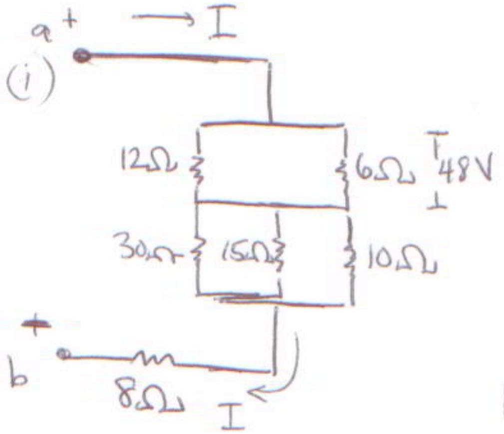
Saturday Review Session Solutions

Problem 1 (i)



(a) Determine I:

12Ω, 6Ω resistors are in parallel

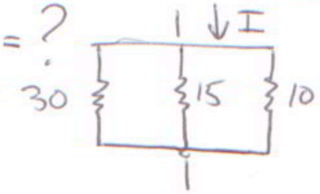
Kirchhoff's 1st law $\Rightarrow \Delta V_6 = \Delta V_{12} = 48\,V$

$I_6 = \frac{48V}{6\Omega} = 8A$, $I_{12} = \frac{48V}{12\Omega} = 4A$

Kirchhoff's 2nd law $\Rightarrow I = I_6 + I_{12} = 12\,A$

(b) $\Delta V_8 = ?$ $\quad \Delta V_8 / 8\Omega = I_8 = I = 12\,A \Rightarrow \Delta V_8 = 96\,V$

(c) $\Delta V_{10} = ?$



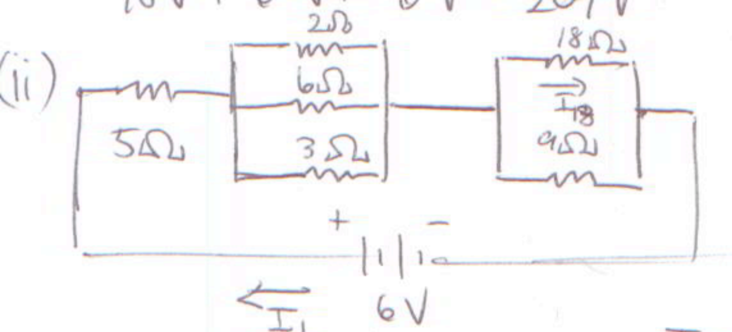
$$R_{eq}^{-1} = \frac{1}{30\Omega} + \frac{1}{15\Omega} + \frac{1}{10\Omega} = \frac{6}{30\Omega}, \; R_{eq} = 5\Omega$$

$$\Delta V_{eq} = \Delta V_{10} = (12A)(5\Omega) = 60\,V$$

(d) $\Delta V_{ab} = ?$ Kirchhoff's 1st law $\Rightarrow \Delta V_{ab} = \Delta V_6 + \Delta V_{10} + \Delta V_8$

$= 48V + 60V + 96\,V = 204\,V$

(ii)



$I_1 = ?$, $I_{18} = ?$

$$R_{eq} = 5\Omega + \left(\frac{1}{2\Omega} + \frac{1}{6\Omega} + \frac{1}{3\Omega}\right)^{-1} + \left(\frac{1}{9\Omega} + \frac{1}{18\Omega}\right)^{-1} = 12\Omega$$

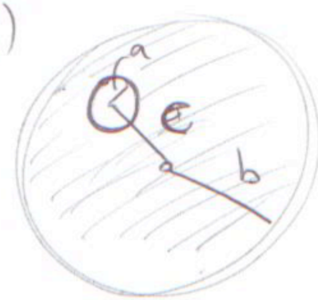
$$I_1 = 6V/12\Omega = \underline{0.5\,A}$$

$I_1 = I_{18} + I_9$; $\Delta V_{18} = \Delta V_9 \rightarrow (18\Omega) I_{18} = (9\Omega) I_9$; $I_{18} = \frac{1}{2} I_9$

$I_1 = I_{18} + 2I_{18} = 3I_{18} \rightarrow I_{18} = 1/6\,A$ //

Problem 2: Ampere's Law

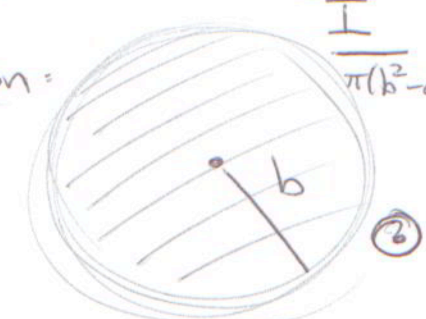
(i)



$\odot$ I Current density $J = \frac{I}{\pi(b^2-a^2)}$. Find $|\vec{B}|$ at axis of the rod.

Use superposition:

$\frac{I}{\pi(b^2-a^2)} = J$



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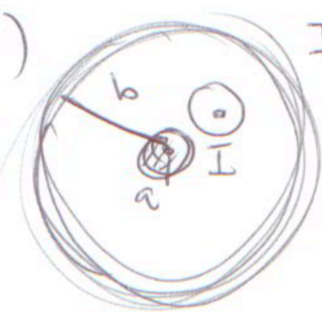
By Ampere's law, magnetic field at the axis of the rod is 0 in the first case. For the second, Ampere's law gives us $|\vec{B}| = \frac{\mu_0}{2\pi c} \frac{I}{\pi(b^2-a^2)} \cdot \pi a^2$

(ii)



$I \otimes$ Find $|\vec{B}|$ for $a<r<b$, $r>b$

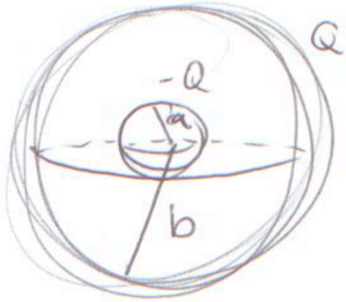
By symmetry, $\vec{B}$ winds around current.

Integrating over an Amperian loop,

$\oint \vec{B} \cdot d\vec{l} = |\vec{B}(r)| 2\pi r = \mu_0 I_{enc}$. If $r>b$, $I_{enc} = 0 \Rightarrow |\vec{B}| = 0$

If $a<r<b$, $I_{enc} = I \Rightarrow |\vec{B}(r)| = \frac{\mu_0 I}{2\pi r}$

Problem 3: Spherical Capacitor



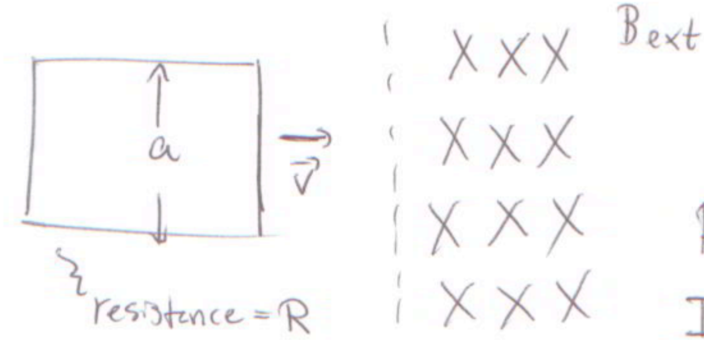
Potential for spherical shell (uniformly charged)

$$V(r) = \begin{cases} -\frac{1}{4\pi\epsilon_0}\frac{Q}{r} & r > R \\ -\frac{1}{4\pi\epsilon_0}\frac{Q}{R} & r < R \end{cases}$$

Potential obeys superposition: potential at outer shell = $\frac{1}{4\pi\epsilon_0}\left(-\frac{Q}{b}+\frac{Q}{b}\right)=0$, potential at inner shell = $\frac{1}{4\pi\epsilon_0}\left(-\frac{Q}{b}+\frac{Q}{a}\right)$

$\Rightarrow \Delta V = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$, $C = \frac{Q}{\Delta V} = \frac{4\pi\epsilon_0}{\left(\frac{1}{a}-\frac{1}{b}\right)}$

Problem 4: Induction

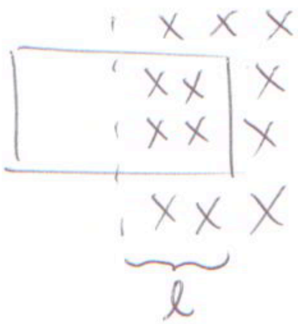


What force does the loop experience?

$\Phi_B = -B_{ext}\, a\, l \Rightarrow -\frac{d\Phi_B}{dt} = B_{ext}\, a\, v$

Faraday $\Rightarrow \mathcal{E}_{emf} = B_{ext}\, a\, v = IR$

$I = \frac{B_{ext}\, a\, v}{R}$. Lenz's law $\rightarrow$ current is counterclockwise



$\vec{F}$ on top/bottom equal and opposite

$\vec{F}$ on right-hand side $= I\vec{L}\times\vec{B}$

$|\vec{F}| = \left(\frac{B_{ext}\, a\, v}{R}\right) a\, B_{ext}$, force directed opposite to velocity