Solutions

Electromagnetic Waves

a) From the argument of the electric field $(ky-\omega t)$ we know the wave is propagating in the positive y direction. Thus

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$$

must also point in the $+\hat{y}$ direction. This requires $\vec{B}$ to point in the $\hat{x}$ direction. $E_0 = cB_0$ for all EM plane waves so finally

$$\boxed{\vec{B} = \frac{E_0}{c}\sin(kx-\omega t)\,\hat{y}}$$

b) The intensity is the time average of $|\vec{S}|$. Here,

$$|\vec{S}| = \frac{E_0 \cdot E_0/c}{\mu_0}\sin^2(kx-\omega t)$$

The average of $\sin^2(x)$ over a period is $\frac{1}{2}$. I is therefore

$$I = \frac{E_0^2}{\mu_0 c}\cdot\frac{1}{2} = \frac{(10^{-6}\,\text{V/m})^2}{(4\pi\cdot 10^{-7}\,\frac{\text{N}}{\text{A}^2})(3\cdot 10^8\,\text{m/s})}\cdot\frac{1}{2}$$

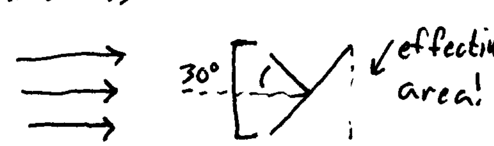
$$= 10^{-13}\cdot\frac{1}{24\pi}\,\frac{\text{W}}{\text{m}^2}$$

$$\boxed{I = 1.3\cdot 10^{-15}\,\frac{\text{W}}{\text{m}^2}}$$

c) i) This is just intensity times area, $\boxed{P = 1.3\cdot 10^{-15}\,\text{W}}$

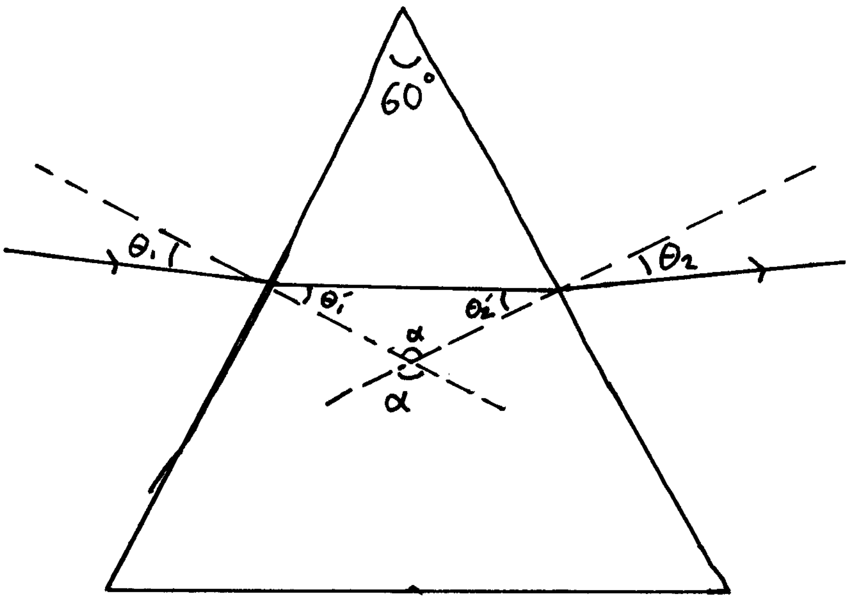
ii) This time, the actual area the wave hits is $1\,\text{m}^2\cdot\cos 30^\circ$ so $\boxed{P = 1.1\cdot 10^{-15}\,\text{W}}$

↑ see figure to right →



Refraction

a) When $\theta_1 = \theta_2$, we have the following



It must be that $\theta_1' = \theta_2'$.

The angle α is

$360° - 60° - 90° - 90° = 120° = \alpha$

So $\theta_1' = \theta_2' = 30°$.

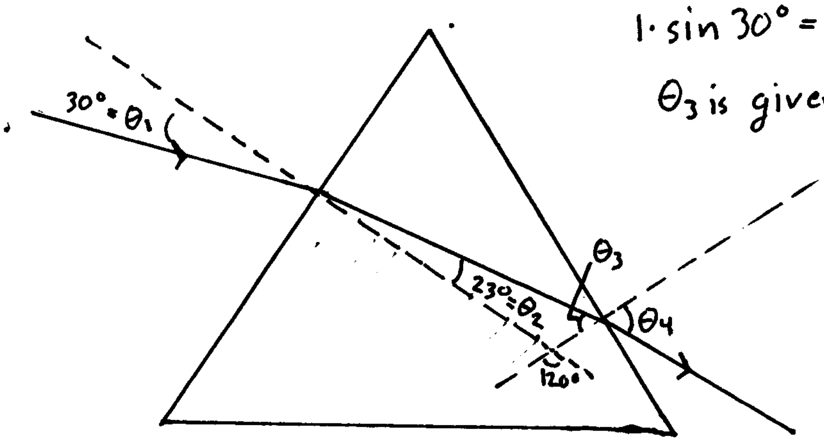
We then solve Snell's law

$1 \cdot \sin\theta_1 = 1.3 \sin 30°$

$\rightarrow \theta_1 = \arcsin(1.3 \cdot \sin 30°)$

$\boxed{\theta_1 = 40.5°}$

b) Since we are given θ_1, we just follow the ray using Snell's law.



$1 \cdot \sin 30° = 1.3 \sin\theta_2 \rightarrow \theta_2 = 23°$

θ_3 is given by $180° - 120° - 23° = 37°$

Then $1.3 \sin 37° = \sin\theta_4$

$\rightarrow \boxed{\theta_4 = 51°}$ (This is θ_2 in the statement of the problem)

Contact Lens

The situation is that we have two lenses at essentially the same location. Applying the thin lens formulas to the first lens gives

$$\frac{1}{f_c} = \frac{1}{d_{o,c}} + \frac{1}{d_{i,c}}$$

where $d_{o,c}$ is the original object's distance and $d_{i,c}$ is the image distance of the contact lens.

Since the lenses are very close, $d_{i,c} = -d_{o,e}$. The second lens equation is therefore

$$\frac{1}{f_e} = \frac{1}{d_{o,e}} + \frac{1}{d_{i,e}} = \frac{1}{d_{o,c}} - \frac{1}{f_c} + \frac{1}{d_{i,e}}$$

which can be written as

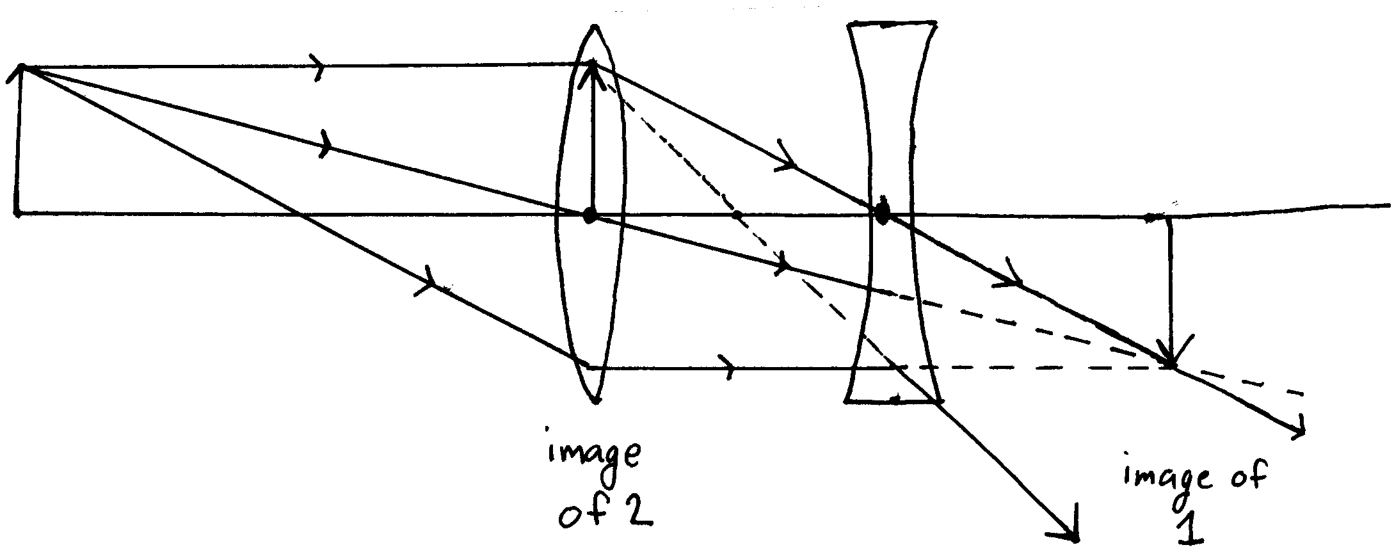
$$\frac{1}{f_e} + \frac{1}{f_c} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f_{eff}}$$

($d_o = d_{o,c}$ is the distance of the object from the effective lens; $d_i = d_{i,e}$ is the same for the image)

So $f_{eff} = \left(\frac{1}{f_e} + \frac{1}{f_c}\right)^{-1} = \boxed{\frac{f_e f_c}{f_e + f_c} = f_{eff}}$

Multiple Lenses

a)



b)

For the first lens $d_o = 2f = 2d$, $f = +d$

$$\frac{1}{f} = \frac{1}{2f} + \frac{1}{d_i} \rightarrow d_i = 2f = 2d$$

This means $d_o = -d$ for the second lens, and $f = -d/2$

$$\frac{1}{-d/2} = \frac{1}{-d} + \frac{1}{d_i} \rightarrow d_i = -d$$

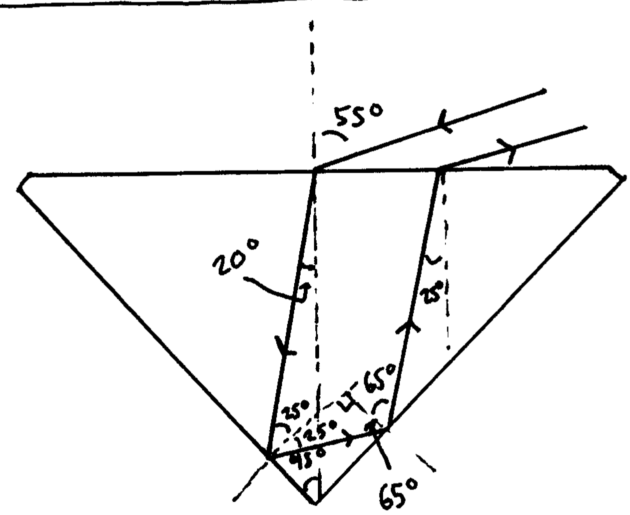
(This is in agreement with my ray tracing)

Calculate M for each lens:

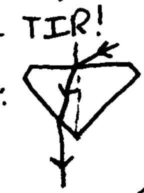
$$M_1 = \frac{-2d}{2d} = -1 \qquad M_2 = \frac{-(-d)}{-d} = -1$$

Then $M_{Total} = (-1) \cdot (-1) = 1$

Refraction and TIR (in a diamond)



b) If this were glass, θ_c would be larger and the ray would not have TIR! Roughly like this:



<u>How to draw this</u>

1) Use Snell's law for the first interface $(1 \cdot \sin 55° = 2.4 \cdot \sin 20°)$
2) Calculate $\theta_c = \arcsin\left(\frac{1}{2.4}\right) = 24.6°$
3) Use geometry and notice we have TIR for the next two interfaces
4) Use law of reflection twice $(\theta_I = \theta_R)$
5) Since all the inside angles add to $180°$ $(25° + 25° + 65° + 65°)$, the last ray is parallel to the incident ray