

1) (10 points)

Unpolarized microwaves (radio waves) are moving left to right along the z axis with an intensity I. They hit a barrier made of a large number of closely spaced horizontal wires; call the direction of these wires the x axis.

Combination

a) What are the directions of the E and B fields of the microwaves after passing through these wires?

Calculation

a) What are the peak magnitudes of the E and B fields in the microwaves after passing through the wires?

A) unpolarized light has its electric field \vec{E} pointing in any of the possible polarization directions. Those are any direction \perp to the direction of motion \hat{z} . But E fields in the \hat{x} and \hat{y} directions cause different effects in the wires. E along \hat{x} will cause charges to move along the wires, and that will cancel out the x component of \vec{E} . E along \hat{y} can't move charges between wires, so isn't cancelled. So after going through the wires, \vec{E} is along \hat{y} (up). Since \vec{E} & \vec{B} are \perp to each other and \perp to the direction of motion (\hat{z}), \vec{B} must be along \hat{x} .

B) After the polarizer, only I/2 of intensity is still there. Intensity is related to the RMS E field via

$$I/2 = \frac{1}{c\mu_0} E_{\text{rms}}^2 = \frac{1}{c\mu_0} \left(\frac{E_p}{\sqrt{2}}\right)^2$$

$$\text{so } E_{\text{peak}} = c\mu_0 I$$

but note that (from Poynting vector & definition of intensity in EM wave) $\frac{1}{\mu_0} EB = \frac{1}{c\mu_0} E^2$ $E = B/c$

$$B_{\text{peak}} = cE_{\text{peak}} = c^2\mu_0 I$$

2) (15 points)

You have an object 2cm high. You want to create an inverted, real image of it 30cm to the right by placing a lens in between. The image is to be 4cm high.

Idea

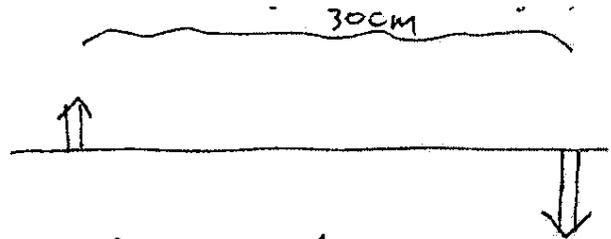
a) What type (concave, convex) of lens will you use? Why?

Idea

b) By drawing rays, show approximately where you'll put the lens and that it forms a real image

Calculation

c) Find the focal length of the lens and the distance it's placed from the object, in centimeters



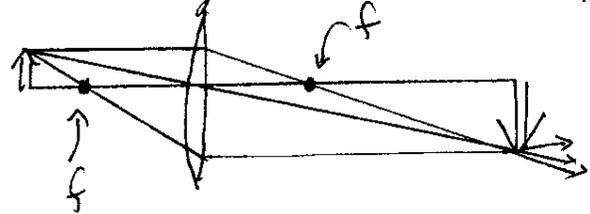
a) light rays coming from the object need to be focused to a point at the image - only a converging lens is needed to do that

b) The ray through the center of the lens is unbent, so draw a straight line:



Now add two more rays to show image forms in right place

since light goes past image, image is real



c) for image to be twice as high as object, image distance must be twice as large as object distance. Since they sum to 30cm, the object distance is 10cm

$$\text{Then } \frac{1}{10} + \frac{1}{20} = \frac{1}{f} \quad \frac{3}{20} = \frac{1}{f} \quad f = \frac{20}{3} \approx 6.66 \text{ cm}$$

which looks consistent with the drawings

3) (15 points)

You have a camera that cannot focus on an object that's closer than 60cm from the front of the lens. You focus on a flower at 60cm distance, and move in until the front of the lens is 30 cm from the flower.

- Idea
Calculation
Combination
- a) The image is blurry; draw a diagram and explain why.
b) You now put one lens right in front of the camera lens to fix the blurriness. What type of lens (concave or convex) and what focal length should you use?
c) After fixing the blurriness the image on the camera's film larger, the same size as, or smaller than it was when the camera was 60cm from the flower? In other words, have you changed the size of the image? Explain.



The image forms behind the film. Rays from one point on the flower hit many points each on the film, blurring it out

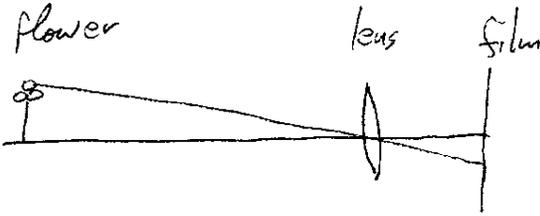
- b) Must make object (flower) at 30 cm appear to be at 60 cm, so camera can focus on the image
- object distance 30 cm
image distance -60 cm

$$\frac{1}{30} + \frac{1}{-60} = \frac{1}{f}$$

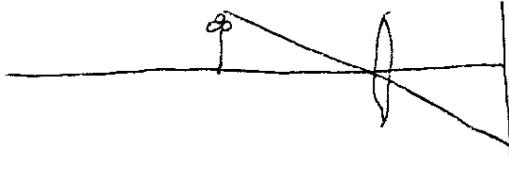
$$f = 60 \text{ cm} > 0, \text{ so } \underline{\text{convex}}$$

- c) see over

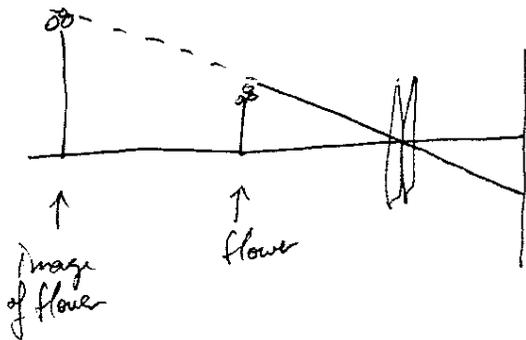
3c) Consider 3 cases, & look at central ray



flower @ 60 cm - This is original size on film



flower @ 30 cm - flower closer, so angle larger, image on film larger (but blurry!)



flower @ 30 cm, but image back at 60 cm. Not blurry, but also same angle as 2nd case above → image on film is larger than when flower was @ 60 cm in case 1!

So this is a magnifying glass, a way to get larger pictures.

Can also be done using the magnification formula for the camera lens

$$|m| = \left| \frac{\text{image distance}}{\text{object distance}} \right|$$

← lens → film distance, a constant
 ↙ 60 cm originally, 60 cm finally

Size on film = $|m|$ (size of camera object)

↙ virtual image is twice as big ⇒ twice as large!

4) (10 points)

On a sunny day, the pupil of your eye has a diameter of 4mm. Assume that the lens of your eye is perfect, and that the diameter of your eyeball is 2.5cm. If needed, you can assume that your near-point of vision is at 30cm.

Calculation

a) If you are looking at a true point source of light of 660 nm wavelength, what's the smallest spot that the lens can form on your retina? ~~Define~~

Combination

b) Using that spot size, find the closest two points that you can resolve. Be sure to explain your reasoning. Define resolution

a) Due to diffraction, light spreads out over an angular range $\theta = \pm 1.1 \frac{\lambda}{D}$

$\leftarrow 660\text{nm} = 660 \times 10^{-7}$
 $\leftarrow \text{pupil size } 4\text{mm} = 4 \times 10^{-3}$

$= \pm 1.8 \times 10^{-4} \text{ radians}$

Now convert to spot on retina



(2.5cm)

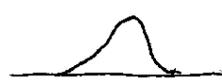
total angle = $2 \times 1.8 \times 10^{-4}$ (was $\pm 1.8 \times 10^{-4}$)

$= 2.5 \times 10^{-2} \times 1.8 \times 10^{-4}$

$= 4.5 \times 10^{-6} \text{ meters}$

$= 4.5 \text{ microns}$

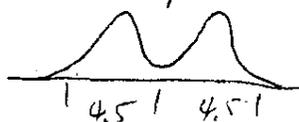
b) ~~so~~ so a point looks like this



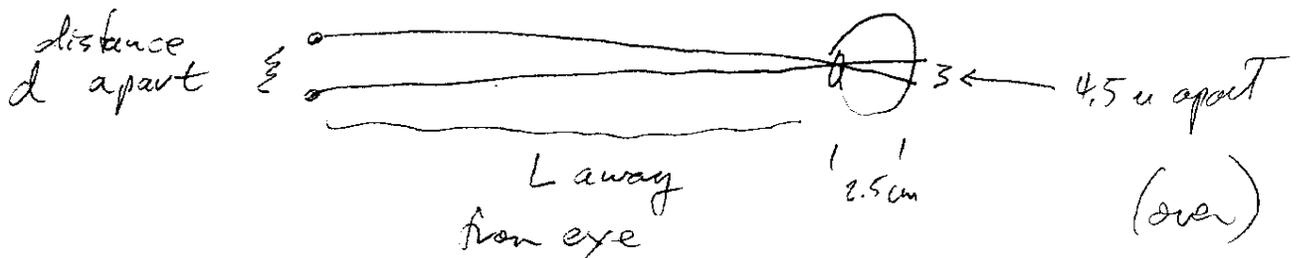
on the retina

4.5 μ

Call two points 4.5 μ apart on the retina the definit of resolution



Then those two points ^{4.5} have to be seen:



4b) Similar triagh

$$\frac{d}{L} = \frac{4.5 \times 10^{-6}}{2.5 \times 10^{-2}} = 1.8 \times 10^{-4} \text{ radians}$$

That makes sense - its just the angle that light is diffracted out by going through pupil!

But what's L ? If we want to maximize resolution, eg smallest possible d , want smallest possible L . To say another way, to see things clearly, we bring them close. Closest possible clear vision is "near point."

$$\frac{d}{30 \times 10^{-2}} = 1.8 \times 10^{-4}$$

$$\begin{aligned} d &= 30 \times 10^{-2} \times 1.8 \times 10^{-4} \\ &= 54 \times 10^{-6} \text{ meters} = 54 \text{ microns} \end{aligned}$$

which seems about right