Midterm II Review Problems

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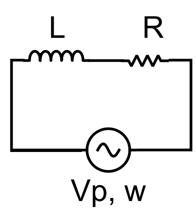
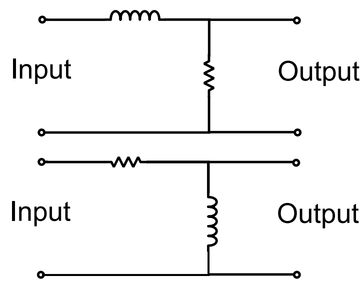
1 High and Low Pass Filters

Consider the following circuit, consisting of an inductor of inductance L and a resistor R in series with an AC power supply (peak voltage V_p, frequency ω). What is the peak current I_p in the circuit? What are the peak voltages across the inductor, $V_{p,L}$ and across the resistor, $V_{p,R}$? How do these peak voltages vary with the frequency ω (consider the two cases $\omega \to 0$ and $\omega \to \infty$)?



Now consider connecting the inductor and resistor to some signal with a large mix of frequencies and amplitudes instead of a simple power supply. If we want to eliminate all of the high frequency components of the signal, and only keep the low frequencies, in which configuration should we connect the output? Why? How about vice-versa (keeping the high-frequency components)?



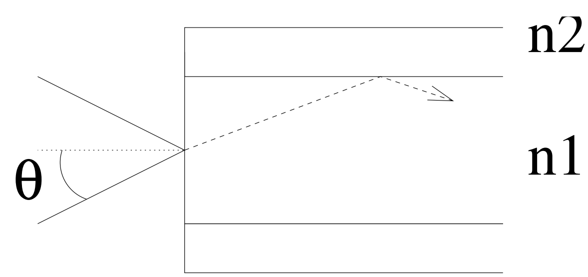
Finally, how could we create similar devices with a resistor and a capacitor (from what we know of how capacitors behave at high/low frequencies)?

2 Light Waves and Intensity

The total power output of the sun is approximately $4 * 10^{26}$ W (!). What is the intensity of the solar radiation at the surface of the sun? What are the magnitudes of the electric and magnetic fields in sunlight? What is the total power of the sunlight falling on the Earth? What is its intensity? What is the radiation pressure on the Earth due to sunlight, and what is the total force due to this pressure (assume all of the sunlight is absorbed by the earth). Some useful constants: $R_{sun} = 7.0 * 10^8$ m, $R_{earth} = 6.4 * 10^6$ m, 1AU(radius of Earth's orbit) $= 1.5 * 10^{11}$m

3 Optical Fiber Acceptance Angle

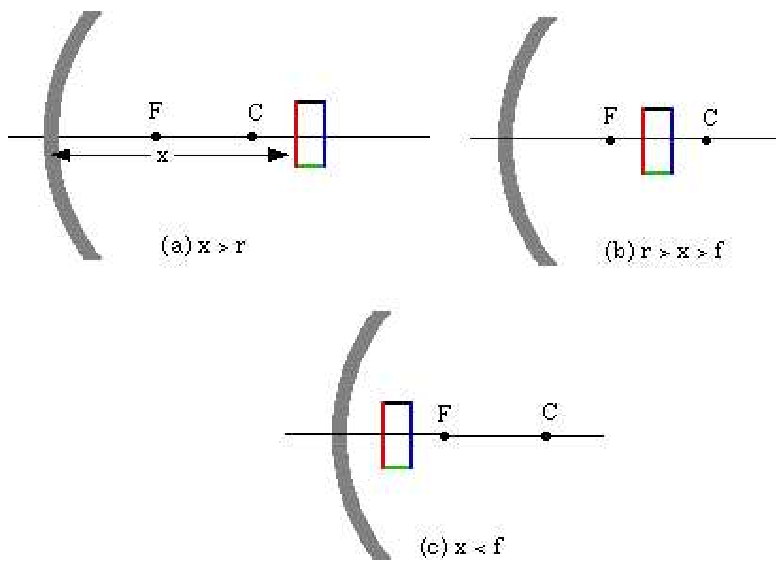
As we've seen in lab, an optical fiber can transmit light via total internal reflection. A simple model for an optical fiber consists of a core material with index of refraction n_1 and cladding (surrounding material) with index of refraction n_2 (how should n_1 compare with n_2 to create a working optical fiber?).



When we want to use the fiber, we can shine light in at the end - but the angle at which we shine it will matter, as we'll see in this problem. As a function of n_1 and n_2, what is the maximum angle θ (from the fiber) at which we can shine a beam of light and expect it to be to be transmitted along the fiber? Try and simplify your answer as much as possible. What happens when we go to greater angles?

4 Mirrors w/ 3D Objects

We've discussed (and seen) lens and mirror arrangements with 1D/2D objects, but what do things look like with objects that have some depth? For the concave mirror and rectangular object below, what shape and size are the images? Where are they positioned? What if the mirror were convex? Can you analyze similar setups with lenses?



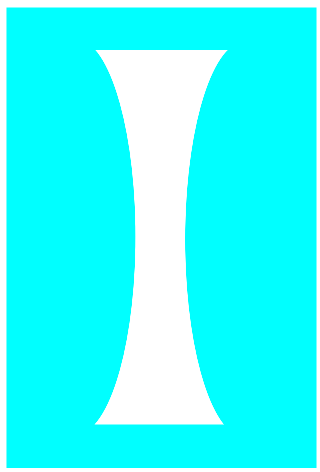
5 Lensing with Water

The lensmaker's formula that we normally use is simplified for the case of a lens in air, but we can use a more general form to consider lenses in different surrounding media such as water ($n = 1.33$):

$$\frac{1}{f} = \left(\frac{n_{lens}}{n_{surrounding}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{1}$$

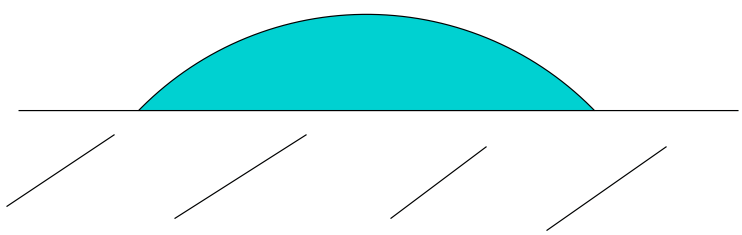
In particular, take a lens with focal length +10 cm in air, made out of glass with index of refraction $n = 1.53$. What is its focal length underwater?

Now that you've got your SCUBA gear on to test out your answer, you take a look through a bubble you've made that's shaped like a diverging lens:



Assuming the bubble stays this shape, demonstrate with a convincing diagram whether this lens is diverging or converging. Then, use the lensmaker's equation presented above to find its focal length, given $R_1 = R_2 = 5cm$.

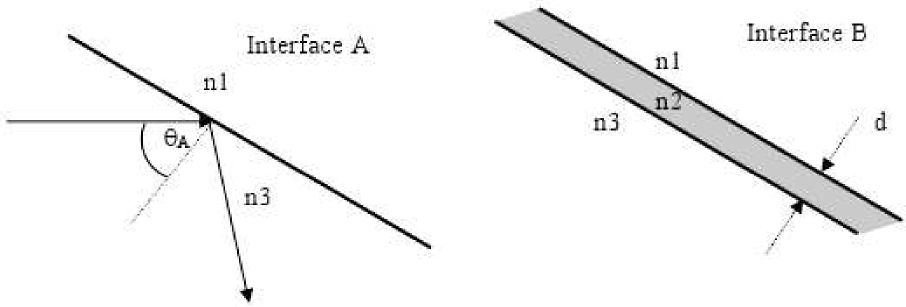
Finally, consider a drop of water, $n = 1.33$, with radius of curvature 50 cm on top of a flat mirrored surface.



What is the focal length of this setup, and where is the image formed for an object held 60 cm from the mirror's surface?

6 TIR at an Interface

Consider the two interfaces drawn below, with $n_1 < n_2 < n_3$.



Interface A is between two materials whereas interface B has an extra intermediate layer between the two of uniform thickness d. We want to compare the critical angles for the two interfaces, i.e. the angle at which ALL of the light is reflected and none is transmitted. Let θ_A be the critical angle for interface A, and find the critical angle for interface B, θ_B, in terms of θ_A, n_1, n_2, n_3 and d.