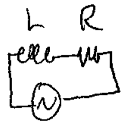
1. (1



$$Z = i\omega L + R$$

$$|Z| = \sqrt{Z^* Z} = \sqrt{(R+i\omega L)(R-i\omega L)} = \sqrt{R^2+\omega^2 L^2}$$

Then, use Ohm's Law: $V = IZ$

$$V_p = I_p \cdot |Z| = I_p \sqrt{R^2+\omega^2 L^2}$$

$$\boxed{I_p = \frac{V_p}{\sqrt{R^2+\omega^2 L^2}}}$$

Then,

$$V_{p,L} = I_p \cdot |Z_L| = I_p \cdot \omega L \Rightarrow \boxed{V_{pL} = \frac{V_p \omega L}{\sqrt{R^2+\omega^2 L^2}}}$$

Likewise,

$$V_{p,R} = I_p \cdot |Z_R| = I_p R \Rightarrow \boxed{V_{p,R} = \frac{V_p R}{\sqrt{R^2+\omega^2 L^2}}}$$

For $\omega \to 0$, $V_{p,L} \to 0$ and $V_{p,R} \to V_p$

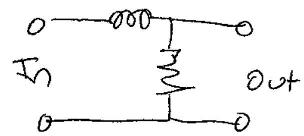
For $\omega \to \infty$, $V_{p,L} \to V_p$ and $V_{p,R} \to 0$

So, for low frequencies, if we measure voltage across the inductor, we see nothing ("no signal"), but all of the voltage instead is across the resistor.

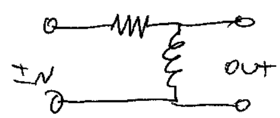
Vice-Versa for high frequencies.

(2

Then, if we want to use this as a filter, to keep the low frequencies (low-pass) we connect the output across the resistor:



For a high-pass filter, we connect the output across the inductor:

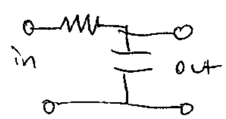


Capacitors are the opposite of inductors in a sense - they transmit high frequencies well, but impede low frequency signals.

As filters, we could use:

High Pass

Low Pass



$\boxed{2}$ $Power_{Sun} = 4 \times 10^{26}\,W$

Intensity $S_{sun} = \dfrac{P_{sun}}{Area} = \dfrac{P_{sun}}{4\pi R_{sun}^2} = \dfrac{4 \times 10^{26}\,W}{4\pi \cdot (7 \times 10^8\,m)^2} = \boxed{6.5 \times 10^6 \,\frac{W}{m^2}}$

For the fields,

$$S = \frac{E_p^2}{2\mu_0 c} = \frac{cB_p^2}{2\mu_0} \Rightarrow E_p^2 = 2\mu_0 c S \;\; \& \;\; B_p^2 = \frac{2\mu_0}{c} S$$

This gives

$$E_p = 7.0 \times 10^4 \, V/m \qquad B_p = 2.3 \times 10^{-4}\, T$$

At the Earth's orbit,

$$S_{\text{Sun, orbit}} = \frac{P}{4\pi R^2_{\text{earth orbit}}} = \frac{4\times10^{26}\,W}{4\pi\,(1.5\times10^{11}m)^2} = \boxed{1.4\times10^3\ W/m^2}$$

The Earth's cross-sectional area is $\pi R_e^2 = \pi\,(6.4\times10^6)^2 = 1.3\times10^{14}\,m^2$

So the total power falling on the earth is

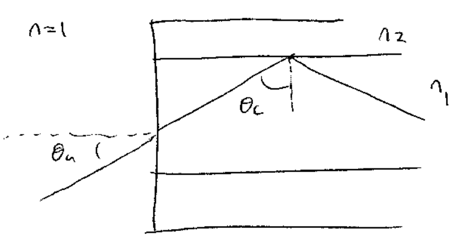
$$P_{\text{earth}} = S_{\text{orbit}}\,\pi R_e^2 = 1.4\times10^3\,W/m^2 \cdot 1.3\times10^{14}\,m^2 = \boxed{1.8\times10^{17}\ W}$$

Radiation pressure is

$$\frac{S}{c} = \frac{1.4\times10^3\,W/m^2}{c} = \boxed{4.7\times10^{-6}\ N/m^2}$$

Total force is Pressure $\cdot$ Area $= 4.7\times10^{-6} \cdot 1.3\times10^{14} = \boxed{6.1\times10^8\ N}$

$\boxed{3}$

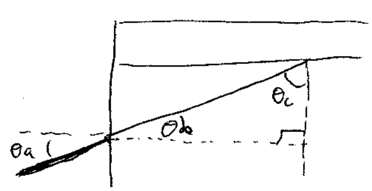


$n_1 > n_2$

The angle θ_a is such that when light goes into the material n_1, it hits the edge at the critical angle and gets totally reflected.

If we are above θ_a, then some light will get transmitted when it hits the cladding and ultimately the light won't get kept in the fiber.

So:



$$\sin\theta_c = \frac{n_2}{n_1}$$

$$\theta_b + \theta_c + 90° = 180° \Rightarrow \theta_b + \theta_c = 90°$$

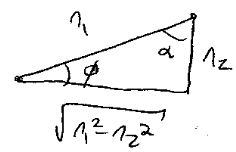
Using Snell's Law,

$$1 \cdot \sin\theta_a = n_1 \sin\theta_b$$

$$= n_1 \sin(90 - \theta_c)$$

$$\boxed{\sin\theta_a = n_1 \sin\left(90 - \sin^{-1}\left(\frac{n_2}{n_1}\right)\right)}$$

We can simplify this:



$$\sin^{-1}\left(\frac{n_2}{n_1}\right) = \phi$$

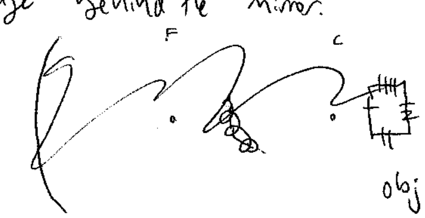
$$90 - \sin^{-1}\left(\frac{n_2}{n_1}\right) = \alpha$$

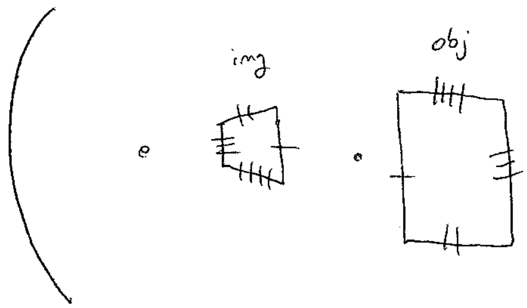
$$\sin\alpha = \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$$

So

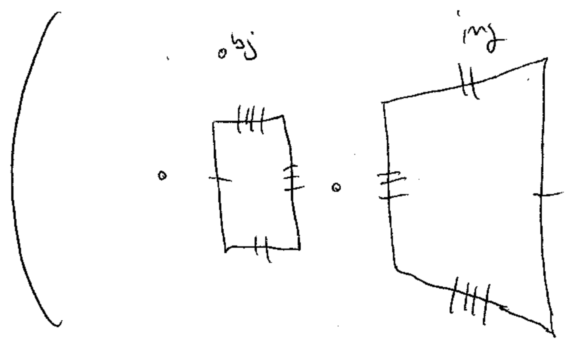
$$\boxed{\sin\theta_a = \frac{n_1 \cdot \sqrt{n_1^2 - n_2^2}}{n_1} = \sqrt{n_1^2 - n_2^2}}$$

$\boxed{4}$ We can approach these problems by considering what happens to the near and far sides. I won't use any numbers, but just the facts that something <u>outside</u> the center C gives a <u>real image</u> inside C, something between F and C gives a real image outside C, and something inside F gives a virtual image behind the mirror.

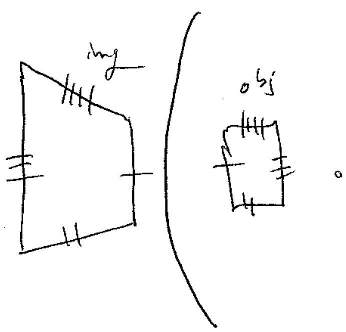




img is small, inverted, real



img is inverted, large, real



Img is upright, large, virtual

$\boxed{5}$ In air, $f = +10\text{cm}$, $n = 1.53$, so:

$$\frac{1}{10} = (1.53 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Underwater,

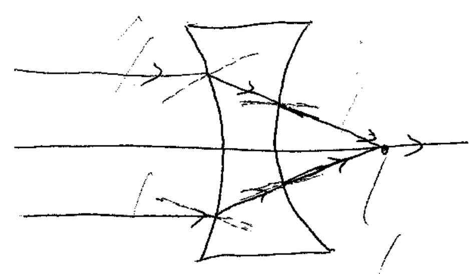
$$\frac{1}{f} = \left(\frac{1.53}{1.33} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

We can solve for $\frac{1}{R_1} - \frac{1}{R_2}$ from the first eqn: $\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{10} \cdot \frac{1}{(0.53)} = 0.189$

Then,

$$\frac{1}{f_{water}} = \left(\frac{1.53}{1.33} - 1\right)\left(0.189\right) = 0.0284 \Rightarrow \boxed{f_{water} = 35\,\text{cm}}$$

So $f_{water} > f_{air}$.

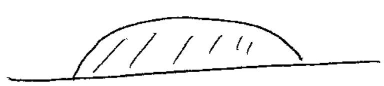


Converging

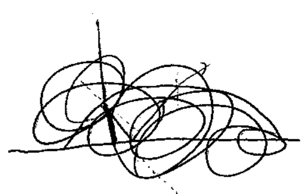
$$\frac{1}{f} = \left(\frac{1}{1.33} - 1\right)\left(\frac{-1}{5} - \left(\frac{1}{-5}\right)\right) = (-0.25)\cdot\left(\frac{-2}{5}\right)$$

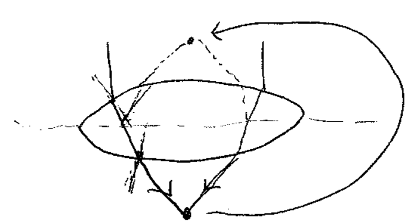
$$= 0.1$$

So $f = +10$ cm



We can treat this system as one complete lens, and then reflect back in the mirror:





So, this is effectively one lens, double convex, w/ $R_1 = R_2 = 50$c

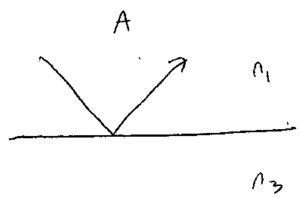
$$\frac{1}{f} = (1.33-1)\left(\frac{1}{50}+\frac{1}{50}\right) = 0.0132 \Rightarrow \boxed{f = 75.8}$$

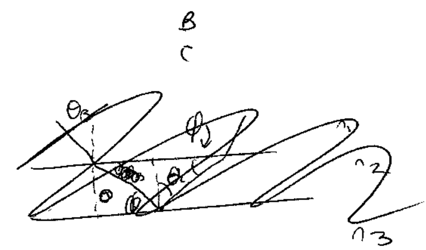
Then, putting something @ 60 cm gives an image distance:

$$\frac{1}{f} = \frac{1}{S_{img}} + \frac{1}{S_{obj}} \Rightarrow S_{img} = -288\text{ cm}$$

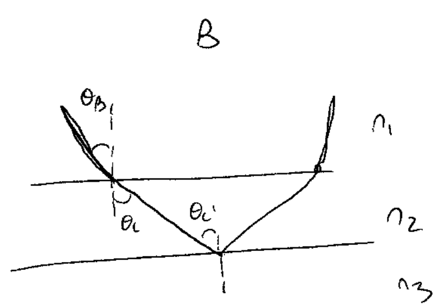
So the lens produces an image (virtual) 288 cm ~~above the~~ behind it (on the side light is coming from). Flipping this around in the mirror gives an image (still virtual) 288 cm below the mirror.

6





$\theta_A = \sin^{-1}(n_3/n_1)$



For B, again, we want θ_B to give a refracted ray that hits the second interface ($n_2 \to n_3$) at the critical angle.

~~$n_1 \sin\theta_B = n_2 \sin\theta_c$~~

90°

~~$\theta_c = \sin^{-1}(n_3/n_2)$~~

~~So, $\sin\theta_B = \frac{n_2}{n_1}\sin($~~

$$n_1 \sin\theta_B = n_2 \sin\theta_c$$

$$\sin\theta_c = \frac{n_3}{n_2}$$

So,

$$n_1 \sin\theta_B = \frac{n_3}{n_2} \cdot n_2 = n_3$$

So, $\theta_B = \sin^{-1}\left(n_3/n_1 \right) = \theta_A$

(Very similar to #3...)