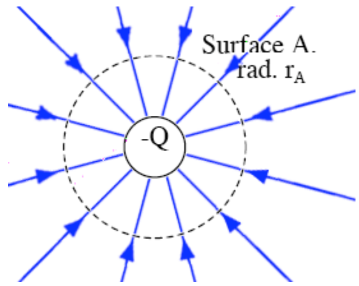
8B Worksheet Answers

3. Gauss' Law and Electric Fields

1. a)



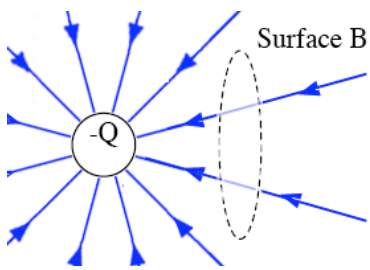
b) Negative - the lines are *entering* the surface.

c) $\Phi_A = -Q / \varepsilon_0$

d) $|E(\text{surface A})| = \dfrac{-Q}{4\pi\varepsilon_0 r_A^2}$

Note that surface B is a closed ellipsoid surface (like the shell of an egg), *not* a disk, which is an open surface.

e)

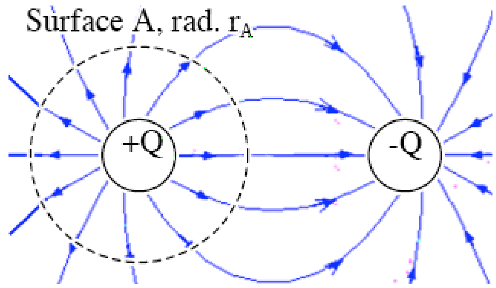


f) Zero - every field line that enters surface B also leaves surface B.

g) $\Phi_B = 0$

h) No - the electric field will vary depending on how far from the charge we are, as Coulomb's Law says. The *flux* of the electric field through *all* of surface B, however, is zero [the electric field would be zero if *no* lines entered or left the surface - saying the flux is zero amounts to saying that the number leaving minus the number entering is zero.]

2. a)

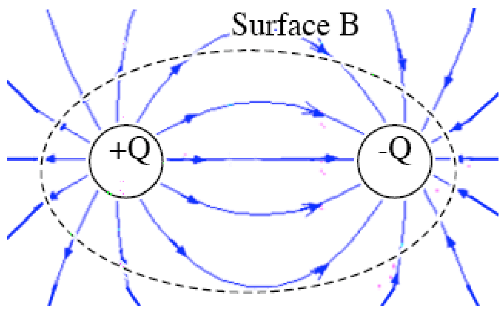


b) Positive - the lines are *leaving* the surface.

c) $\Phi_A = +Q / \varepsilon_0$

d) No - the contribution from just charge $+Q$ has that form, but the contribution from the negative charge will change the electric field.

e)



f) Zero - every field line that leaves surface B also enters surface B.

g) $\Phi_B = 0$.

h) No, by the same reasoning as part (h) from problem 1.

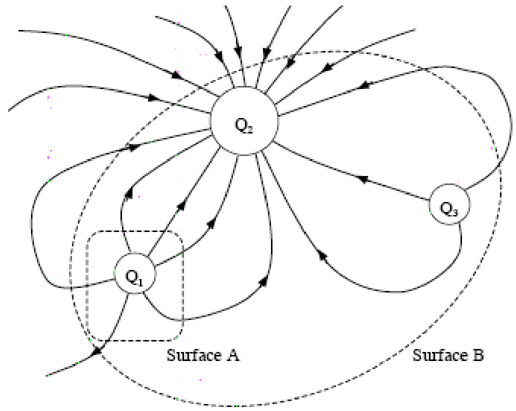
3. a) *FALSE*: Gauss' Law only states that the *net flux* of the electric field through the *whole* surface is 0 in this case.

b) *FALSE*: Gauss' Law states that the *net* charge enclosed by the surface is zero.

c) *FALSE*: The flux of the *E*-field only depends on the charge enclosed. We can imagine an imaginary cube with a point charge outside of it. There is no net flux through the cube, but there certainly is a non-zero electric field at all points on the imaginary surface!

d) *FALSE*: Gauss' Law is one of the four Maxwell's equations for electromagnetism and is *always* valid. The reason we care about symmetry relates to the *usefullness* of *using* Gauss' Law to *find* the *E*-field based on a charge distribution [the symmetry lets us reduce the ugly surface integral of the dot product of the *E*-field and area vectors into a simple *EA*.]

4. a)



b) Conversion: 6 '*lines*' => 12Q ==> 1 '*line*' => 2Q
$Q_I = +12Q$; $Q_2 = -30Q$; $Q_3 = +6Q$

c) $\Phi_B = -12Q / \varepsilon_0$

d) Yes: We can see that 3 lines 'leave' surface B while 9 lines 'enter' surface B, giving a net flux of -6 lines. A flux of +6 lines gave Φ_A for surface A, so the flux Φ_B should be equal in magnitude and opposite in sign as Φ_A, which it is!

5. a) *FALSE*: E only is guaranteed to be zero at places where the conductor actually is. For instance, E will *not* be zero inside the cavity where the $+Q$ charge is.

b) *TRUE*

c) *FALSE*: To guarantee that there is no net electric field inside the conducting material, a net charge of $-Q$ has to reside on the surface of the cavity containing the $+Q$ charge.

d) *TRUE*

6. a) $Q_\oplus \approx 6.1 \times 10^5$ Coulombs

b) A distance 19110 km from the *center* of the Earth, or a distance 12740 km above the *surface* of the Earth.

c) *Weaker*! The charge enclosed by an imaginary gaussian sphere of a radius *less* than Earth's radius will increase as r^3 as r increases. The area of the gaussian surface only increases as r^2, however. This means that as r increases, the electric field goes like $Q/A \sim r$. The electric field strength increases linearly as we go away from the Earth's center to the surface and then, after we pass the surface of the Earth, decreases as $1/r^2$ [since now the charge enclosed is fixed, rather than increasing like r^3.] The maximum value of the electric field, then, is *at* the surface of the Earth!

7. The charge distribution in this problem is *spherically symmetric*, so the Electric field has the form $\vec{E} = \frac{Q_{enc}(r)}{4\pi\varepsilon_0 r^2}\hat{r}$ at all points. $Q_{enc}(r)$ is the net amount of charge enclosed by an imaginary sphere of radius r, centered at the origin, r is the radial distance from the origin and $\hat{r}$ points radially outward from the origin.

   a) At r = 0.5 m, $Q_{enc}(r)$ = +2 C, so the E-field points radially *outwards*.

   b) $|E(r = 0.5 \text{ m})| = 7.2\times 10^{10}$ N/coul (2 coulombs is a *very* large amount of charge!)

   c) At r= 1.5 m, we are inside the metal shell, which is a *conductor*. Therefore, the E-field must be zero!

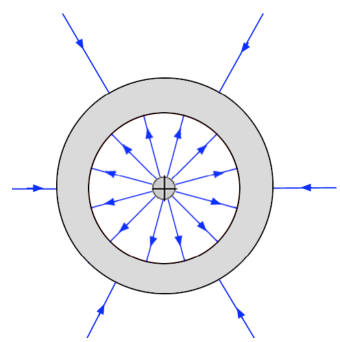
   d) At r = 2.5 m, $Q_{enc}(r)$ = -1 C, so the E-field points radially *inwards*.

   e) $|E(r = 2.5 \text{ m})| = 1.4\times 10^{9}$ N/coul

   f) From our answer to part (c), $E(r = 1.5 \text{ m}) = 0$, so $Q_{enc}(r = 1.5 \text{ m}) = 0$. The only place a net charge can live on a conductor in an electrostatic situation is on the surfaces of the conductor, so $Q_{enc} = +2\text{C} + Q_{\text{inner surface}} = 0$, so $Q_{\text{inner surface}} = -2$ C.

   g) The conductor has a net charge of -3 C, so $Q_{\text{outer surface}} = -1$ C.

   h)



8. a) $|E(r = R)| = 0$ b) $|E(r = 3R)| = \frac{\lambda}{2\pi\varepsilon_0 R}$ c) $\vec{F} = \frac{\lambda q_e}{\pi\varepsilon_0 R}\hat{x}$

9. a) III > I = II > IV ($E_{\text{I}} = E_{\text{II}} = 3/2\, E_{\text{III}} = 2\, E_{\text{IV}}$)

   b) IV > II > III > I ($E_{\text{I}} = 0$; $E_{\text{II}} / 2 = E_{\text{III}} = E_{\text{IV}} / 3$)

   c) The electron will hit the right plate (with $+2Q$ of charge on it)

   d) $\Delta t = \sqrt{\frac{3m_e dA\varepsilon_0}{2q_e Q}}$