8B Worksheet Answers

5. Capacitance

1. a) $C = \varepsilon_0 A / d$ b) $V_C = V$

   c) $Q = CV = \varepsilon_0 AV / d$ d) $U = CV^2 / 2 = \varepsilon_0 AV^2 / 2d$

   e) The plate separateion, d increases, and A and Q are held fixed. Q is held fixed, rather than V, because the plates are no longer connected to the battery, or to anything else, so the net charge on one of the plates can't move to the other plate. Therefore, C decreases by a factor of 3, Q stays the same, $V_C = Q / C$ increases by a factor of 3. For U we have to be careful - we can't just take d and make it $3d$, since the d came from the expression for the charge, which is now fixed. Therefore, we will take $U = Q^2 / 2C$ as our expression. Q is fixed, and C decreases, so U increases by a factor of 3.

   f) The increase in stored energy U comes from the fact that we have to add energy into the system to move the plates away from each other. That is, the plates are oppositely charged and thus feel an attractive force towards each other. We have to do work on the system to move the charged plates *away* from each other, and this added work is converted into energy stored in the electric field of the capacitor (we have increased the volume over which the electric field is non-zero by futher separating the places.

2. abcd) Same as 1a, 1b, 1c, and 1d!

   e) The plate separation, d decreaases, and A and V are held fixed. V is held fixed, rather than Q, because the plates remain connected to the battery, which forces the two plates to maintain a potential difference of V when things have reached steady state. Therefore, C increases by a factor of 2, V_C stays the same, and $Q = CV_C$ increases by a factor of 2. For U, we will use the expression $U= CV^2 / 2$, since V is the quantity we want to keep fixed. Therefore, U increases by a factor of 2.

   f) The extra charge comes from the fact that we are still hooked up to the battery, which will take more charge from the already negative plate and push it over to the already positive plate, making the negative plate more negative and the positive plate more positive.

3. a&b) $C_{eq,i} = C$ $Q_{net,i} = CV = 2.4 \times 10^{-5}$ coul

   $C_{eq,ii} = 3C$ $Q_{net,ii} = 3CV = 7.2 \times 10^{-5}$ coul

   $C_{eq,iii} = 2/3\ C$ $C_{net,iii} = 2/3\ CV = 1.6 \times 10^{-5}$ coul

c) There are now 'more places' to put the charge. If C is already charged up, and we add $2C$ in parallel, we have just added another path for current to take, so current will now start to flow through the battery again to create a charge distance on the new capacitor, which increases the total amount of charge you can store for a given potential difference!

d) Imagine that the top plate of the C capacitor is positive. The middle arrangement of the bottom plate of the C capacitor, the top plate of the $2C$ capacitor, and the connecting wire is initially neutral, and *isolated*, so the net charge will always be zero. The positive charges on the top plate of C and the negative charges on the bottom plate of $2C$ 'push' and 'pull' electrons from the bottom plate of the middle segment to the top plate of the middle segment, leaving the net charge zero and creating a charge separation - up to the point that there will be a net charge $-Q$ on the top plate (i.e. the bottom plate of the C capacitor) and a net charge $+Q$ on the bottom plate (i.e. the top plate of the $2C$ capacitor). The charges on $C$and $2C$ are therefore equal in size.

e) $V_{\text{left}} = V = 12\text{V}$ $\qquad$ $V_{\text{right}} = V = 12\text{V} = V_{\text{left}}$
$Q_{\text{left}} = CV = 2.4 \times 10^{-5}$ coul $\qquad$ $Q_{\text{right}} = 2CV = 4.8 \times 10^{-5}$ coul

f) $Q_{\text{top}} = 2/3\ CV = 1.6 \times 10^{-5}$ coul $\qquad$ $Q_{\text{bottom}} = 2/3\ CV = 1.6 \times 10^{-5}$ coul
$V_{\text{top}} = Q_{\text{top}} / C = 8$ V $\qquad$ $V_{\text{top}} = Q_{\text{bottom}} / 2C = 4$ V
Note that the total voltage drop across both capacitors, $V_{\text{top}} + V_{\text{bottom}}$, is the same as the voltage across the battery, $V = 12\text{V}$. This is one of Kirchhoff's Rules.

4. a) The two 2 Farad capacitors (which we will label C_{top} and C_{bottom}) can be considered as in series with eachother. If we treat the two rightmost capacitors (the ones in series) as one equivalent capacitor, then that equivalent capacitor can be considered to be in parallel with the 4 Farad capacitor (which we will label C_{left}).

b) $V_{\text{batt}} = V_{\text{left}} > V_{\text{top}} = V_{\text{bottom}} = V_{\text{batt}} / 2$

c) $Q_{\text{left}} = 1.5$ coul $\qquad$ $V_{\text{left}} = Q_{\text{left}} / C_{\text{left}} = 0.375$ V
$Q_{\text{top}} = V_{\text{top}}C_{\text{top}} = 0.375$ coul $\qquad$ $V_{\text{top}} = V_{\text{left}} / 2 = 0.1875$ V
$Q_{\text{bottom}} = Q_{\text{top}} = 0.375$ coul $\qquad$ $V_{\text{bottom}} = V_{\text{top}} = 0.1875$ V
Note that we used $V_{\text{left}} = V_{\text{top}} + V_{\text{bottom}}$, and then used the symmetry of the top and bottom capacitors to say that $V_{\text{top}} = V_{\text{bottom}}$ and thus that $V_{\text{top}} = V_{\text{left}} / 2$.

d) $V_{\text{batt}} = V_{\text{left}} = V_{\text{top}} + V_{\text{bottom}} = 0.375\text{V}$

e) (i) $U = U_{\text{left}} + U_{\text{top}} + U_{\text{bottom}} = 0.281\text{ J} + 0.035\text{ J} + 0.035\text{ J} = 0.351\text{ J}$

(ii) $C_{equiv} = C_{left} + \left(\frac{1}{C_{top}} + \frac{1}{C_{bottom}}\right)^{-1} = 4\text{ Farad} + 1\text{ Farad} = 5\text{ Farad}$

$U = C_{\text{equiv}}V_{\text{batt}}^2 / 2 = 0.351\text{ J}$