UNIVERSITY OF CALIFORNIA, BERKELEY
Physics Department

8B, Liphardt | Spring Term 2011 | **MT2**

Name ______________________

Disc. Section ________________

SID ________________________

#1 ______ (20)
#2 ______ (10)
#3 ______ (15)
#4 ______ (10)
#5 ______ (10)
#6 ______ (10)

Total______ of 75

The relative weight of each problem is indicated next to the problem number.

Show every step of your derivation.

!!!!!!!!!!!GOOD LUCK!!!!!!!!!!!

If you get stuck on one problem, go on to the next and come back to the difficulties later in the exam period. Never, never, never quit!! As usual, before you start to manipulate equations you should think for a moment and make sure that you have a general sense of the important features of a problem. Also, please check answers for the right units. For example, if your answer to any problem is something like 10^{418} Ohm/meters, you should check your work.

Problem 1. Brief questions/answers [20] Maxwell's Equations are able to explain all electromagnetic field phenomena.

(A) Which of Maxwell's Equations is/are most applicable to explain the behavior of a **capacitor**? State the Equation(s) and describe the connection.

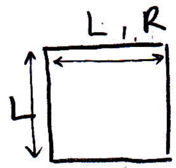
(B) Which of Maxwell's Equations is/are most applicable to explain the behavior of an **inductor**? State the Equation(s) and describe the connection.

(C) Which of Maxwell's Equations is/are most applicable to explain the behavior of **electromagnetic waves**? State the Equation(s) and describe the connection.

(D) Which of Maxwell's Equations is/are most applicable to explain the behavior of the **transformer**? State the Equation(s) and describe the connection.

(E) If magnetic monopoles were observed, which of Maxwell's Equations would be most changed? Explain.

Problem 2 [10]. A wire square with sides L = 10 cm and resistance R = 100 Ohms sits inside a uniform magnetic field of unknown strength. The magnetic field points into the page, and the wire square sits in the plane of the page. Over a time interval of 8 s, the magnetic field decreases by a factor of 4. In that same time interval, the current in the wire is 0.7 A. What is the initial strength of the magnetic field?



$\otimes \vec{B}$

$$I = \frac{V}{R}$$

$$V = -\frac{d\Phi_B}{dt} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{\Delta(BA)}{\Delta t} = -\frac{A(\Delta B)}{\Delta t}$$

$$B' = \frac{1}{4}B_0$$

$$\Rightarrow \Delta B = B' - B_0 = -\frac{3}{4}B_0$$

$$\Rightarrow IR = -\frac{A(-\frac{3}{4}B_0)}{\Delta t} = \frac{3}{4}\frac{A}{\Delta t}B_0$$

$$\Rightarrow \boxed{B_0 = \frac{4}{3} IR \frac{\Delta t}{A}} \qquad (A = L^2).$$

$$\Rightarrow B_0 = \frac{4}{3}(0.7\text{A})(100\,\Omega)\frac{8\text{s}}{(0.01\text{m})^2} = \frac{4}{3}(0.7)(100)\frac{8}{1\times10^{-4}}\text{ T}$$

$$= \frac{4}{3}(0.7)(8)\times 10^6 \text{ T}$$

$$= \frac{2.8}{3}(8)\times 10^6 \text{ T}$$

$$= \frac{22.4}{3}\times 10^6 \text{ T}$$

$$\approx \boxed{7.4\times 10^6 \text{ T}}$$

Problem 3 [15]. A parallel plate capacitor with circular plates and capacitance C is charged to V volts. A switch closes at t = 0s, and the capacitor is discharged through a resistor with resistance R.

a) Why is there a B field in the capacitor? What is the law that applies?
b) What is the **B** field strength ***inside*** the capacitor at radius r as a function of time?
c) What is the **B** field strength ***outside*** the capacitor at radius r as a function of time?

Useful: charge on capacitor:

$$Q = CV_b\left[1 - e^{-t/RC}\right]$$

(a) Capacitor discharges → Q is changing → E is changing

i.e., $\frac{dE}{dt} \neq 0 \Rightarrow \frac{d\Phi_E}{dt} \neq 0 \rightarrow$ B is induced (Ampere's law with Maxwell's correction)

(b) $Q = CV_b(1-e^{-t/RC}) = CV(1-e^{-t/RC})$

$\Rightarrow E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} = \frac{CV}{\varepsilon_0 A}(1-e^{-t/RC})$ $(A = \pi R_c^2$ ← radius of capacitor$)$.

$\Rightarrow \Phi_E = E(\pi r^2) = \frac{CV}{\varepsilon_0 A}\pi r^2(1-e^{-t/RC}) = \frac{CV}{\varepsilon_0}\frac{r^2}{R_c^2}(1-e^{-t/RC})$

$\Rightarrow \underbrace{\oint B.dl}_{B(2\pi r)} = \mu_0\varepsilon_0\frac{\partial\Phi_E}{\partial t} = \mu_0\cancel{\varepsilon_0}\frac{CVr^2}{\cancel{\varepsilon_0}R_c^2}(-e^{-t/RC})(-\frac{1}{RC})$

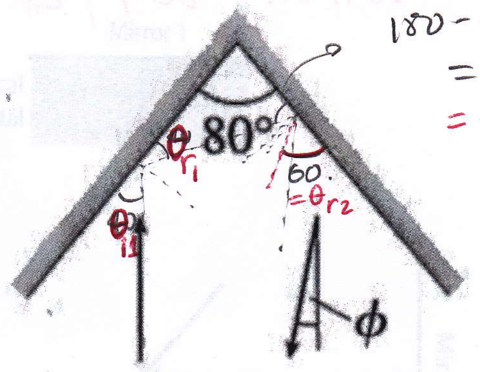
$\Rightarrow B = \mu_0\frac{\cancel{C}Vr}{2\pi R_c^2}\frac{e^{-t/RC}}{R\cancel{C}} = \boxed{\frac{\mu_0 Vr}{2\pi R_c^2}\frac{e^{-t/R}}{R}}$.

(c) $\oint B.dl = \mu_0 I_{enc} + \cancel{\mu_0\varepsilon_0\frac{\partial\Phi_E}{\partial t}}^{0}$ (since E = 0 outside the capacitor).

$B(2\pi r) = \mu_0\frac{dQ}{dt} = \mu_0 CV\frac{d}{dt}(1-e^{-t/RC}) = \mu_0\cancel{C}V\left(\frac{e^{-t/RC}}{R\cancel{C}}\right)$.

$$\boxed{B = \frac{\mu_0 V}{2\pi r R}e^{-t/RC}}$$

Problem 4 [10]. A laser beam is incident on the left mirror. Its initial direction is parallel to a line that bisects the mirrors. What is the angle ϕ of the reflected laser beam?



$180-40-80$
$=60$.
$=\theta_{i2}$

$\varphi = 20°$.

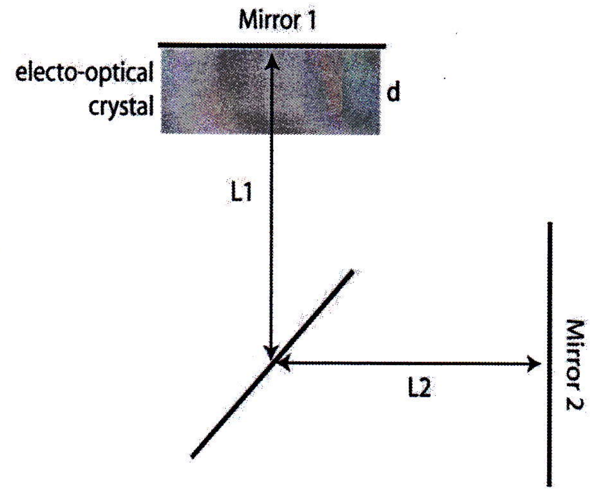
$$\theta_{i1} = \theta_{r1} = 40°$$

$$\theta_{r1} + 80° + \theta_{i2} = 180° \Rightarrow \theta_{i2} = 60°$$

$$\Rightarrow \theta_{r2} = \theta_{i2} = 60°$$

$$\varphi = \theta_{r2} - 40° = 60° - 40° = \boxed{20°}$$

Problem 5 [10]. Interferometers. Electro-optical crystals change their refractive index when a voltage is applied to them. Suppose one arm of a Michelson interferometer contains an electro-optical crystal with thickness d and an initial refractive index n. The interferometer operates with light of wavelength λ. The second mirror is first adjusted to make the output a dark fringe. What is the first refractive index larger than n for which the output is bright?



Start your derivation with this equation: $n = c/v$, where v is the velocity of light, and c is the velocity of light in vacuum.

Method 1:

$$n = \frac{c}{v} = \frac{f\lambda}{f\lambda_n} = \frac{\lambda}{\lambda_n}.$$

$$2d = \frac{1}{2}\lambda_n = \frac{1}{2}\frac{\lambda}{n} \quad \text{(initially)}$$

$$2d = \lambda_{n'} = \frac{\lambda}{n'} \quad \text{(after changing refractive index).}$$

$$\Rightarrow \boxed{n' = 2n.}$$

Method 2:

Initially: $\frac{2d}{\lambda_n}$ = # wavelengths in crystal $= N_0$

After changing n: $\frac{2d}{\lambda_{n'}} = N_0 + \frac{1}{2}$ since we need $\frac{1}{2}\lambda$ to change from destructive interference to constructive interference.

$$\Rightarrow \frac{2d}{\lambda_{n'}} = \frac{2d}{\lambda_n} + \frac{1}{2} \qquad \left(n = \frac{\lambda}{\lambda_n}\right)$$

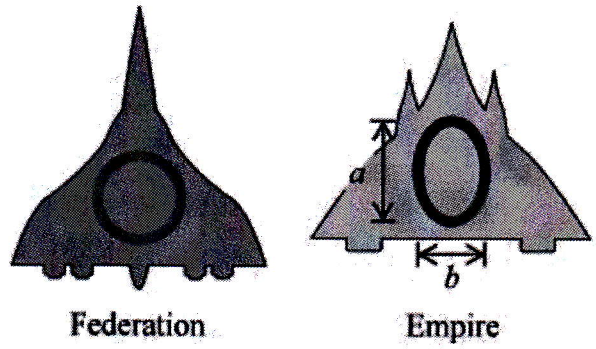
$$\Rightarrow \frac{2n'd}{\lambda} = \frac{2nd}{\lambda} + \frac{1}{2}$$

$$\Rightarrow \boxed{n' = n + \frac{\lambda}{4d}.}$$

Problem 6 [10]. The starships of the Federation are marked with its symbol, a circle, whereas starships of the Empire are marked with its symbol, an ellipse whose major axis is k times its minor axis ($a = kb$) in the figure.

How fast, relative to observers lined up in the direction of a, does an Empire ship have to travel for its markings to be confused with those of a Federation ship?

Use c for the speed of light in a vacuum. Express your answer in terms of k and c.



Want $a' = b$ to have the ellipse turn into a circle.

$$a' = \frac{a}{\gamma} = a\sqrt{1 - v^2/c^2}$$

$a = kb$.

$$\Rightarrow \quad b = kb\sqrt{1 - v^2/c^2} \quad \Rightarrow \quad k\sqrt{1 - v^2/c^2} = 1 \quad \Rightarrow \quad 1 - v^2/c^2 = \frac{1}{k^2}.$$

$$\Rightarrow \quad \frac{v^2}{c^2} = 1 - \frac{1}{k^2}$$

$$\Rightarrow \quad \boxed{v = c\sqrt{1 - 1/k^2} = \frac{c}{k}\sqrt{k^2 - 1}}$$