Physics 8B- Midterm 2 Review Solutions

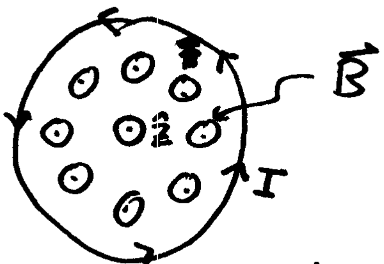
Summer, 2011

1. a) The magnetic field strength for a (long) solenoid with n turns per unit length & carrying a current of I is:

$$|\vec{B}| = \mu_0 n I$$

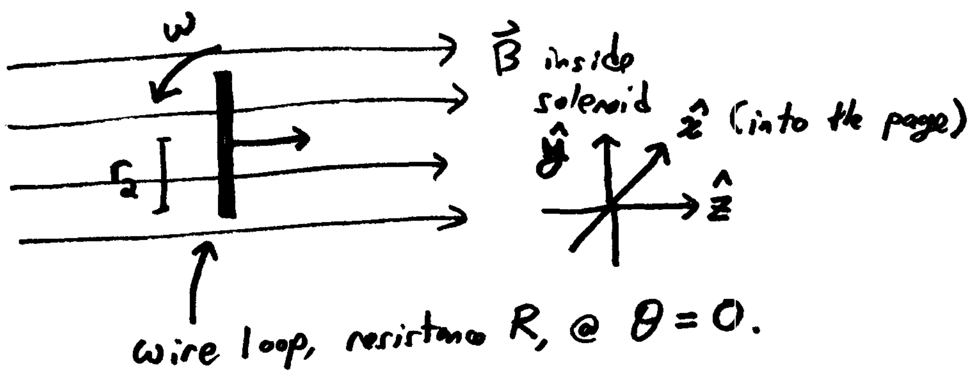
Using the RHR, the field will point along the $+\hat{z}$-axis, so

$$\boxed{\vec{B} = +\mu_0 n I \, \hat{z}}$$

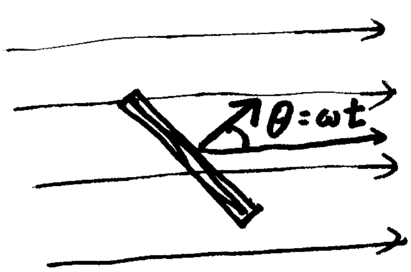


front-view of solenoid

b)



(A normal vector is attached to the loop to more clearly show orientations)



- As the loop rotates, the magnetic flux (due to the solenoid's B-field) changes, inducing an EMF & a current in the loop.

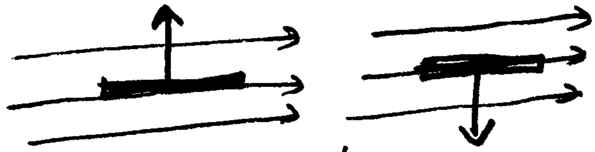
$$\Phi_B = BA\cos\theta = (\mu_0 n I)(\pi r_2^2)\cos\omega t$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -(\mu_0 n I)(\pi r_2^2)\frac{d}{dt}\cos\omega t = \omega(\mu_0 n I)(\pi r_2^2)\sin\omega t$$

$$I = \frac{\mathcal{E}}{R} \Rightarrow I(t) = \frac{\omega}{R}(\mu_0 n I)(\pi r_2^2)\sin\omega t$$

The maximum currents will be when $\sin\omega t = \pm 1 \Rightarrow \theta = 90°, 270°,$ etc.

$$\Rightarrow \boxed{I_{max} = \frac{\omega}{R}(\mu_0 n I)(\pi r_2^2)}$$



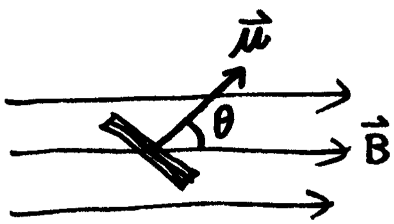
max. current orientation

c) $\vec{\tau}_{net} = \vec{\tau}_{external} + \vec{\tau}_{B\text{-field}} = 0$, since $\omega =$ constant.

$\vec{\tau}_{B\text{-field}}$ is the torque on the loop due to the $\vec{B}$-field,

$\vec{\tau}_{B\text{-field}} = \vec{\mu} \times \vec{B}$, where $\vec{\mu}$ is the magnetic dipole moment vector:

$$|\vec{\mu}| = I_{loop} A_{loop}$$



$$|\vec{\tau}_{B\text{-field}}| = |\vec{\mu}||\vec{B}||\sin\theta|$$

$$= |I_{loop}|(\pi r_2^2)(\mu_0 n I)|\sin\omega t|$$

$$= \underbrace{\frac{\omega}{R}(\mu_0 n I)(\pi r_2^2)\,|\sin \omega t|}_{|I_{loop}(t)|}\,(\pi r_2^2)(\mu_0 n I)\,|\sin \omega t|$$

$$|\vec{\tau}_B| = \frac{\omega}{R}(\mu_0 n I)^2(\pi r_2^2)^2 \sin^2 \omega t$$

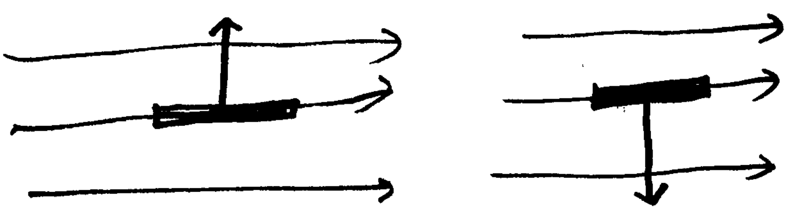
$\Rightarrow$ The magnitude of the external torque must be equal to $|\vec{\tau}_B|$ & point in the opposite direction, so

$$|\vec{\tau}_{ext}| = \frac{\omega}{R}(\mu_0 n I)^2(\pi r_2^2)\underbrace{\sin^2 \omega t}_{max = 1 \text{ @ } \omega t = 90, 270 \text{ etc..}}$$

$$\Rightarrow \boxed{|\vec{\tau}_{ext, max}| = \frac{\omega}{R}(\mu_0 n I)^2(\pi r_2^2)}$$

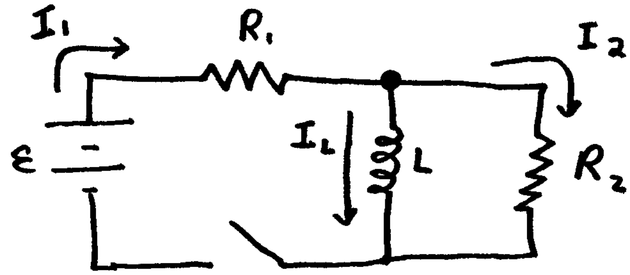
d) The torque applied is maximal when $\sin^2 \omega t = \cancel{\pm 1}\ {+1}$

$\Rightarrow$ $\theta = 90°, 270°$, etc.



orientations of maximal torque!

2.



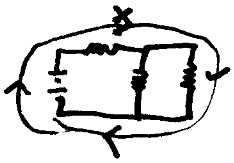
Junction: $I_1 = I_L + I_2$

Before switch closes, $I_1 = I_L = I_2 = 0$.

a) i) <u>Close switch!</u> I_L wants to keep current the same $\Rightarrow I_L = 0$

$\Rightarrow I_1 = I_2$

Loop rule for big loop: $\mathcal{E} - I_1 R_1 - I_2 R_2 = 0$



$$\Downarrow \quad \mathcal{E} - I_2 (R_1 + R_2) = 0$$

$$\Downarrow$$

$$\boxed{I_2 = \frac{\mathcal{E}}{R_1 + R_2}}$$

$I_2 > 0 \Rightarrow$ current flows <u>downward</u>.

ii) After a long time, current through L is <u>constant</u>.

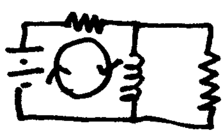
Loop rule for Right loop:

$$\cancel{L \frac{dI_L}{dt}} - I_2 R_2 = 0$$

I_L const. $\Rightarrow \frac{dI_L}{dt} = 0 \Rightarrow \boxed{I_2 = 0} \Rightarrow \cancel{B_1} =$

iii) Continuing from (ii), $I_1 = I_L$

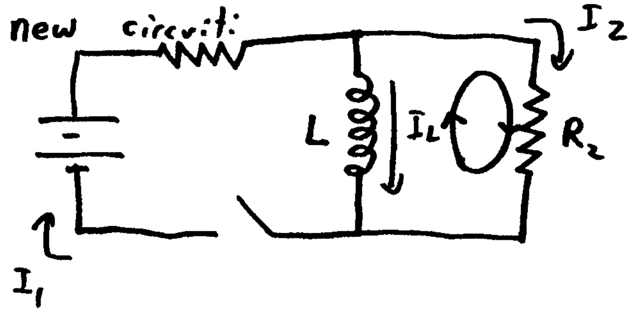
Loop rule for Left loop: $\mathcal{E} - R_1 I_1 - \cancel{L \frac{dI_L}{dt}} = 0$



$$\Rightarrow I_L = I_1 = \frac{\mathcal{E}}{R_1}$$

<u>Open switch</u>

new circuit:



$I_1 = 0$ [broken branch]

$\Rightarrow \quad I_2 = -I_L$

I_L wants to keep current the same $\Rightarrow \quad I_L = \dfrac{\mathcal{E}}{R_1}$

$$\Rightarrow \boxed{I_2 = -\frac{\mathcal{E}}{R_1}}$$

$I_2 < 0 \Rightarrow$ current flows upward.

(iv) After a long time, $I_1 = 0$, $\dfrac{dI_L}{dt} = 0$

$\Downarrow$

$I_2 = -I_L$

$$\Rightarrow \cancel{L\frac{dI_L}{dt}}^{\,0} - I_2 R_2 = 0 \Rightarrow \boxed{I_2 = 0}$$

b) Before the switch reopens, $I_L = \dfrac{\mathcal{E}}{R_1}$, so the energy stored in the inductor is $U_L = \frac{1}{2} L I_L^2$

$$\Rightarrow \underline{U_{L,i} = \frac{1}{2}\frac{L\mathcal{E}^2}{R_1^2}}$$

After a long time with the switch closed, $I_L = 0$, so the energy stored in the inductor is $U_L = \frac{1}{2} L I_L^2 = 0$

$$\Rightarrow \underline{U_{L,f} = 0}$$

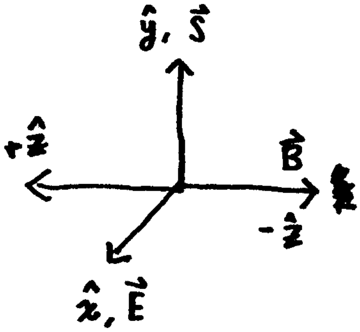
All the energy lost from the inductor was emitted by the resistor, so

$$\boxed{U_{\text{diss by } R} = \frac{1}{2}\frac{L\mathcal{E}^2}{R_1^2}}$$

3. a) $\vec{E} = E_0 \sin(ky - \omega t)\,\hat{x}$

the wave is <u>x-polarized</u>

the wave travels in the <u>$+\hat{y}$-direction</u>



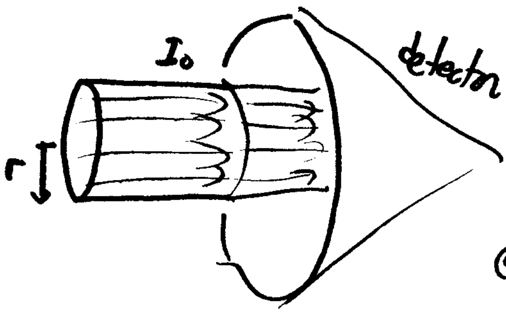
- We need $\vec{E} \times \vec{B}$ to be in the $+\hat{y}$-direction (the direction of propagation)

  $\Rightarrow$ $\vec{B}$ points in the <u>$-\hat{z}$-direction</u>

- $E_0 = cB_0$

$$\Downarrow \quad \boxed{\vec{B} = -\frac{E_0}{c} \sin(ky - \omega t)\,\hat{z}}$$

b) $I_0 = \cancel{\frac{E_0^2}{2c\mu_0}} \ \frac{E_0^2}{2c\mu_0}$



$P_{in} = I_0 A_{beam}$

$= \pi r_{min}^2 I_0$

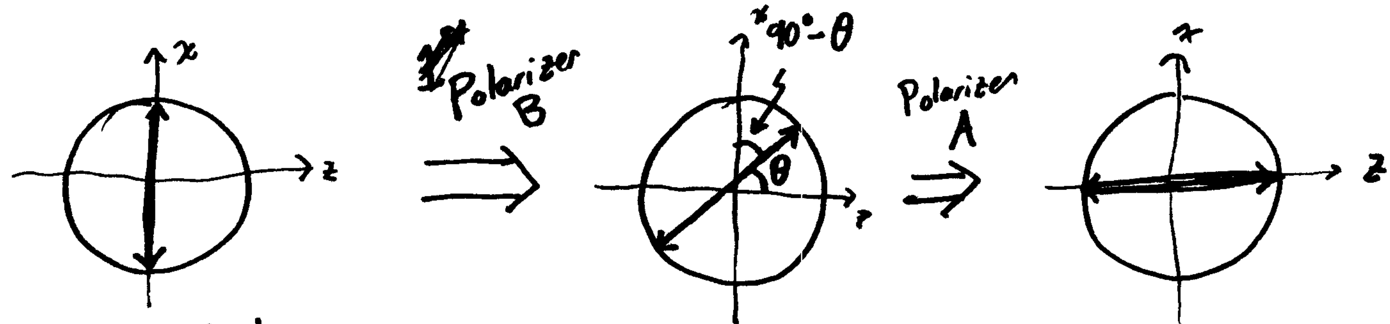
@ minimum r, $P_{in} = P_0$

$\Rightarrow \quad P_0 = \pi r_{min}^2 I_0 = \pi r_{min}^2 \frac{E_0^2}{2c\mu_0}$

$\Rightarrow r_{min}^2 = \frac{2c\mu_0 P}{\pi E_0^2} \quad \Rightarrow \quad \boxed{r_{min} = \sqrt{\frac{2c\mu_0 P}{\pi E_0^2}}}$

c) The laser beam is x-polarized. If ~~the~~ polarizer A completely blocks the beam, then the polarizer axis is perpendicular to the x-axis. Since the beam-axis is in the y-direction, polarizer A must have a polarization axis along the $\underline{\underline{z\text{-direction}}}$.

d)



Light starts out w/ x-polarization

Polarizer A has polarization axis in the z-direction

↳ light emerging from polarizer A is z-polarized

$$I_0 \Rightarrow I_0 \cos^2(90°-\theta) \Rightarrow \left[I_0 \cos^2(90°-\theta)\right]\cos^2\theta$$

$$P_{final} = \left(I_0 \cos^2(90°-\theta)\cos^2\theta\right)A_{beam} = I_0 \pi r_{beam}^2 \cos^2(90°-\theta)\cos^2\theta$$

We know $P_0 = I_0 \pi r_{min}^2$ & $r_{beam} = 2r_{min}$ ↳ $4(I_0 \pi r_{min}^2)\cos^2(90°-\theta)\cos^2\theta$

$$\Rightarrow \boxed{P_f = 4\cos^2(90°-\theta)\cos^2\theta\, P_0} = \left(\sin^2\frac{\theta}{2}\, P_0 \text{ for the trig-adept}\right)$$

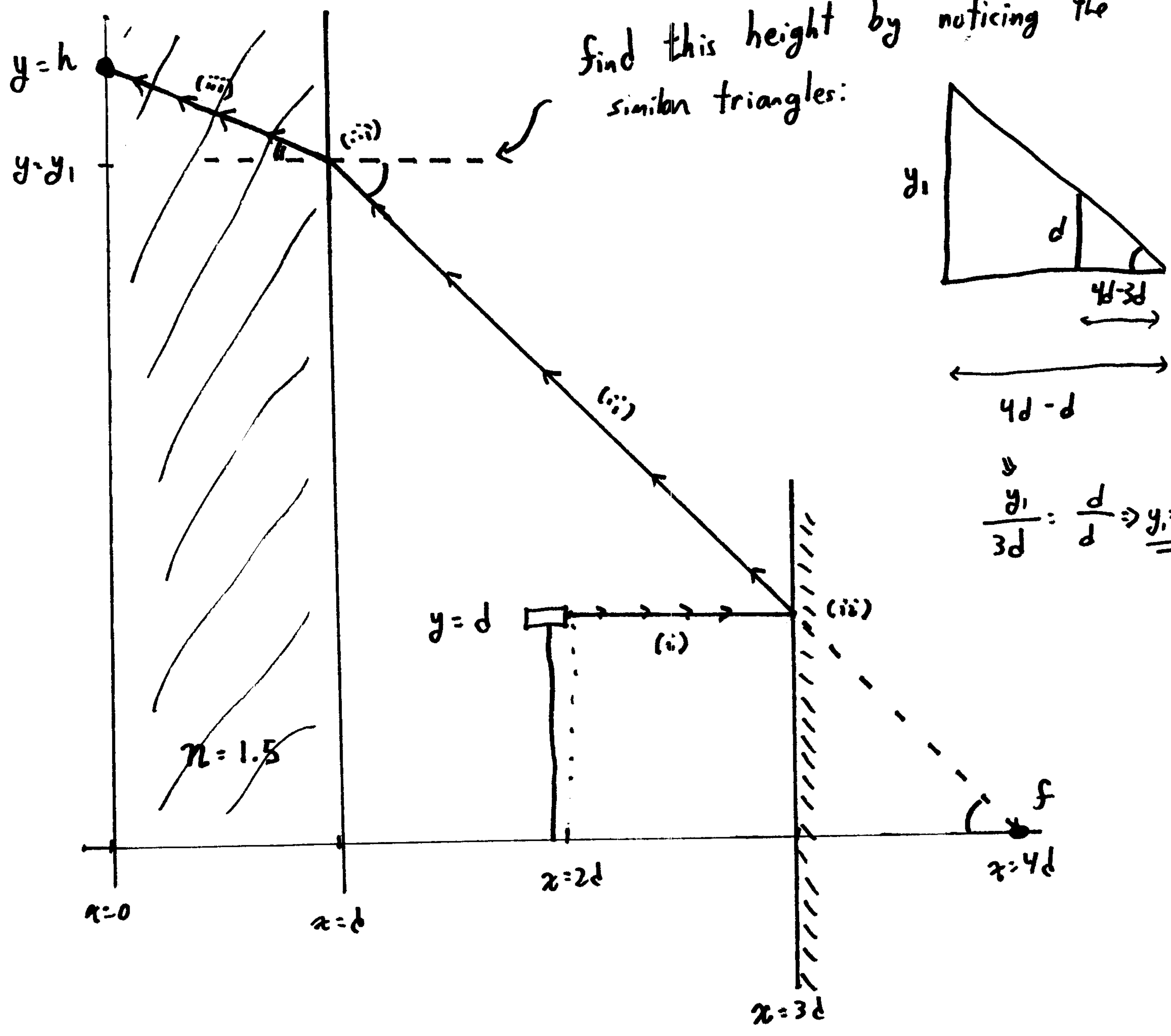
• $\theta = 30° \Rightarrow P_f = 4\left(\frac{1}{2}\right)^2\left(\frac{\sqrt{3}}{2}\right)^2 P_0 = \frac{3}{4}P_0 < P_0 \Rightarrow$ <u>no signal!!</u>

• $\theta = 45° \Rightarrow P_f = 4\left(\frac{\sqrt{2}}{2}\right)^2\left(\frac{\sqrt{2}}{2}\right)^2 P_0 = 4\cdot\frac{1}{2}\cdot\frac{1}{2}P_0 = P_0 \Rightarrow$ <u>signal just turns on!</u>

4. (i) The beam travels parallel to the x-axis at a height of $y = d$.

(ii) It reflects off the mirror & emerges travelling along a line that, when extended backwards, passes through the focal point.

(iii) It hits the boundary between air ($n = 1$) and glass ($n = 1.5$) & refracts, finally hitting the screen.

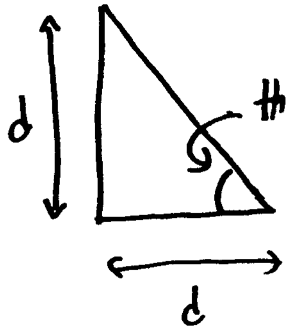


find this height by noticing the similar triangles:

$\Rightarrow$

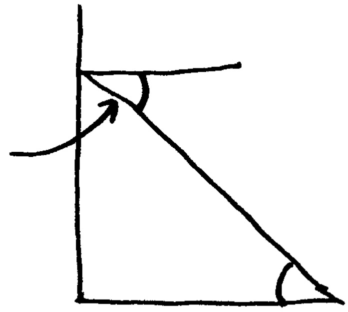
$$\frac{y_1}{3d} = \frac{d}{d} \Rightarrow \underline{\underline{y_1 = 3d}}$$

Also,

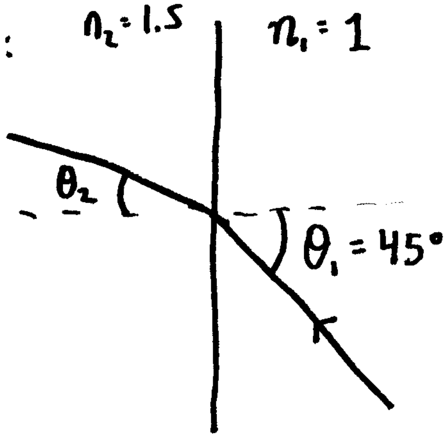


this angle is $45°$

↳ this is also $45°$



Refraction:

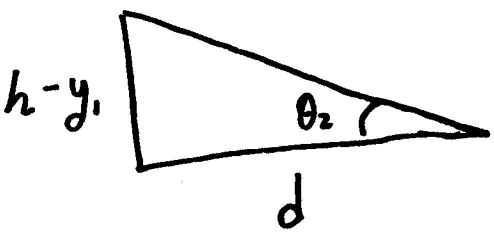


$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\Rightarrow \sin\theta_2 = \frac{\sin 45°}{1.5} = \frac{\sqrt{2}}{3}$$

$$\Rightarrow \underline{\underline{\theta_2 \approx 28.1°}}$$

Now look @ path in glass:



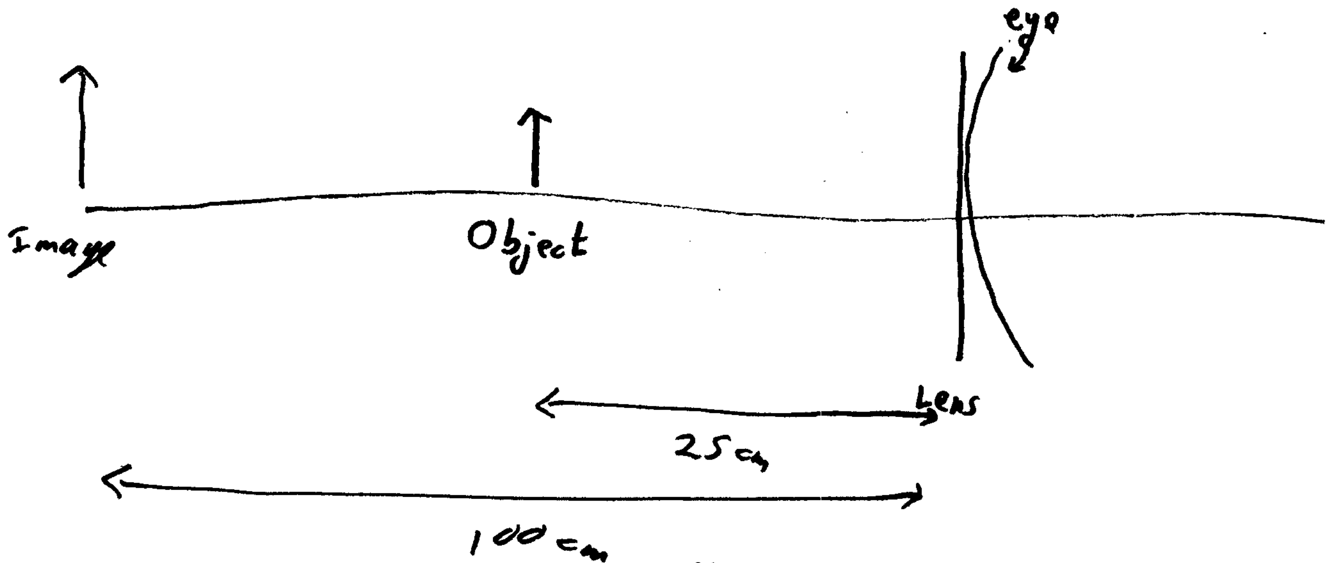
$$\Rightarrow \frac{h - y_1}{d} = \tan\theta_2$$

$$h = d\tan\theta_2 + y_1 = d\tan\theta_2 + 3d$$

$$= 0.53d + 3d$$

$$\Rightarrow \boxed{h = 3.53d}$$

5. The purpose of the lens will be to take an
a) object 25 cm from the lens ~~eye~~ & create an image 100 cm in front of the eye. (The eye can then use this image as a new object and properly focus!)



(not to scale!)

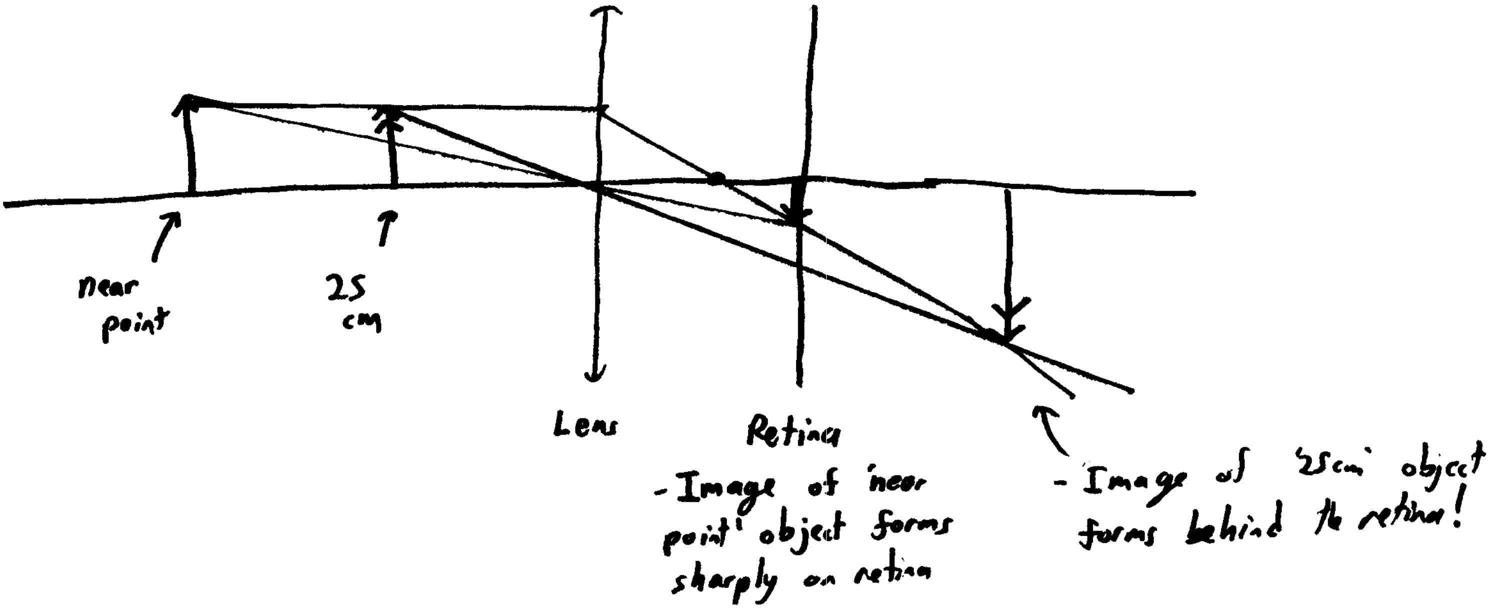
Object in front of lens $\Rightarrow$ $s = +25$ cm

Image in front of lens $\Rightarrow$ $s' = -100$ cm

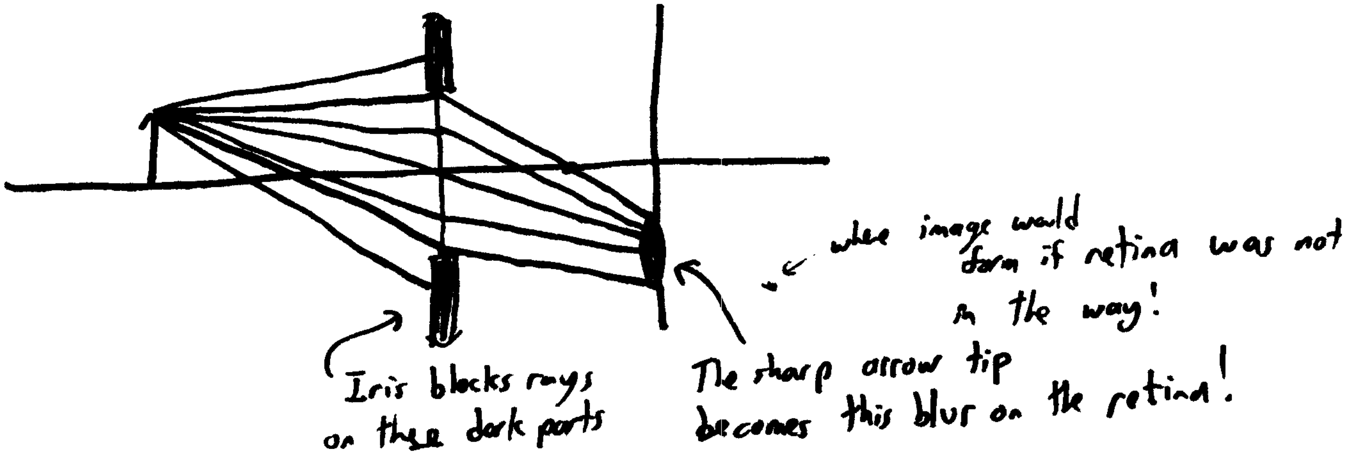
$$\Rightarrow \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{25\text{cm}} - \frac{1}{100\text{cm}} = \frac{3}{100\text{cm}} \Rightarrow \boxed{f = +\frac{100}{3}\text{cm} \approx +33.3\text{cm}}$$

$f > 0 \Rightarrow$ converging lens.

b) Here is a ray diagram for when the eye is focused on the near point of 100 cm & the images formed from two objects, one at $s = 100$ cm one at $s = 25$ cm: (drawing not to scale)



Now look at what happens when the iris is somewhat open:



~~With~~ With pinhole: