

## Worksheet M6 – Solution to selected problems

### Problem 1

(a)  $B = \mu_0 Ni/l$  (notice:  $N/l = n$ )

(e)  $U = \frac{1}{2} LI^2 = \frac{1}{2} (\mu_0 n^2 \pi R^2 l) i^2$

Energy density of magnetic field:  $u = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0} (\mu_0 ni)^2 = \frac{\mu_0}{2} n^2 i^2$

So:  $U = uV = u(\pi R^2 l)$

### Problem 2

Based on the end view picture it's clear that the current is going in straight lines, so you may draw the Amperian loop as a circle concentric with the cylinder (like you would with a Gaussian surface), so that the current does go **through** the loop.

(a)  $B=0$  (net I enclosed =0)

(b)  $B=0$  (no I enclosed)

(c)  $B = \mu_0 i / 2\pi r$ , counterclockwise.

(d)  $\phi = \int_a^b B l dr = \frac{\mu_0 i l}{2\pi} \int_a^b dr/r = \frac{\mu_0 i l}{2\pi} \ln \frac{b}{a}$

### Problem 3

(a) Draw the Amperian loop as a concentric circle inside the wire, that means the enclosed current is only part of the total current  $i$ . Find the current density ( $j = i/A$ ), then integrate over (multiply with) the area of the loop.

$$B(2\pi r) = \mu_0 i \frac{\pi r^2}{\pi R^2}$$

(b)  $u = B^2 / (2\mu_0)$

### Challenge Problem

(a)  $L_1 = \mu_0 n_1^2 l \pi r_1^2, L_2 = \mu_0 n_2^2 l \pi r_2^2$

(b)  $\Phi_2 = n_2 l \phi = n_2 l \left( \frac{L_1}{n_1 l} \right) I_1 = \frac{n_2}{n_1} L_1 I_1$

(c)  $E_2 = \frac{d\Phi_2}{dt} = \frac{n_2}{n_1} L_1 dI_1/dt$

Thus:  $M_{21} = \frac{n_2}{n_1} L_1$

(d) Follow the same procedure, calculate  $\Phi_1$  and  $E_1$ , to get  $M_{12}$