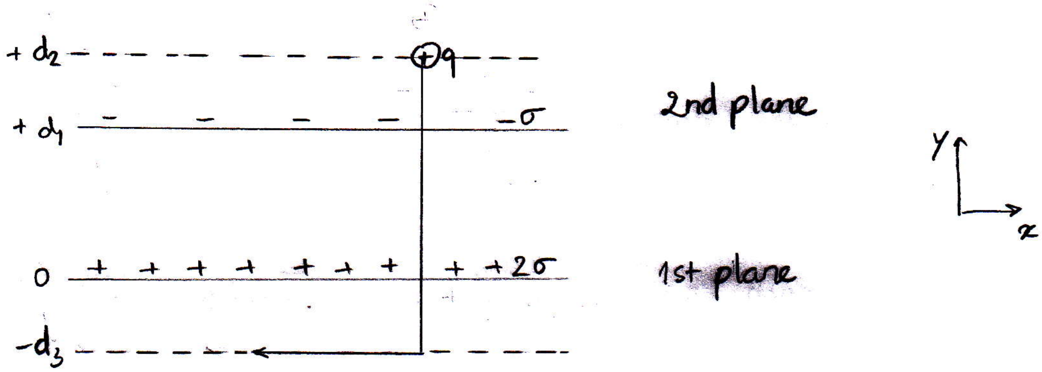
Problem 1:



$$E_{initial} = \frac{Q_{enc}}{2\varepsilon_0 A} = \frac{(-\sigma + 2\sigma)A}{2\varepsilon_0 A} = \frac{+\sigma}{2\varepsilon_0}$$ for any distance d where $d_1 < d < d_2$ above the 1st plane.

$\rightarrow \vec{E}_{initial}$ is in the $+y$ direction

$$E_{between} = -E_{-\sigma} + E_{+2\sigma}$$
$$= \frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} = \frac{3\sigma}{2\varepsilon_0}$$ for any distance d between the 2 planes $(0 < d < d_1)$

$\vec{E}_{between}$ is in the $+y$ direction.

$$E_{final} = \frac{(-\sigma + 2\sigma)}{2\varepsilon_0} = \frac{\sigma}{2\varepsilon_0}$$ in the $-y$ direction for any distance d below the 2nd plane.

Moving a distance L horizontally does not change the potential

$$\Rightarrow \Delta V = -\int_{d_2}^{d_1} E_i\,dr - \int_{d_1}^{0} E_b\,dr - \int_{0}^{-d_3} E_f\,dr$$
$$= -\frac{\sigma}{2\varepsilon_0}(d_1 - d_2) - \frac{3\sigma}{2\varepsilon_0}(0 - d_1) - \left(\frac{-\sigma}{2\varepsilon_0}\right)(-d_3 - 0)$$
$$= \frac{\sigma}{2\varepsilon_0}(d_2 - d_1) + \frac{3\sigma}{2\varepsilon_0} d_1 - \frac{\sigma}{2\varepsilon_0} d_3$$
$$= \frac{\sigma}{2\varepsilon_0}(d_2 - d_1 + 3d_1 - d_3)$$
$$= \frac{\sigma}{2\varepsilon_0}(d_2 + 2d_1 - d_3).$$

$$\Rightarrow \Delta U = q\Delta V = \frac{q\sigma}{2\varepsilon_0}(2d_1 + d_2 - d_3)$$

Problem 2:

Let ρ denote the distance from the cylindrical axis.

Since we have cylindrical symmetry in this problem, we can find the electric field using Gauss's law.
From the cylindrical symmetry, we know that $\vec{E} = E(\rho)\hat{\rho}$.

Choose a Gaussian surface that is a cylinder of radius ρ and length L. Then, by Gauss's Law,

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E(\rho)\,(2\pi\rho L) = \frac{\lambda_{enc} L}{\varepsilon_0}$$

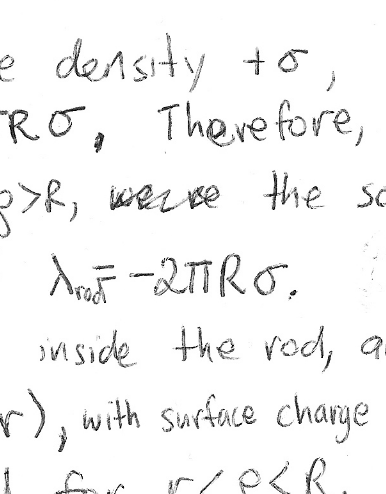
$$E(\rho) = \frac{\lambda_{enc}}{2\pi\varepsilon_0 \rho}$$

For $\rho > R$, we are told that $E(\rho) = 0$ and thus we must have $\lambda_{enc} = 0$.
~~Since~~ The hollow cylinder has surface charge density $+\sigma$, ~~the solid rod must have~~ which corresponds to $\lambda_{cyl} = 2\pi R\sigma$. Therefore, in order to satisfy $\lambda_{enc} = 0$ for the Gaussian surface at $\rho > R$, ~~we see~~ the solid rod must have a charge per unit length of $\lambda_{rod} = -2\pi R\sigma$.
Since the solid rod is a conductor, $\vec{E} = 0$ inside the rod, and all these charges must lie on the surface (at $\rho = r$), with surface charge density $\sigma_{rod} = \frac{\lambda_{rod}}{2\pi r} = -\sigma\frac{R}{r}$.
The final task is to find the electric field for $r < \rho < R$. For the Gaussian cylinder of radius ~~r~~ ρ, with $r < \rho < R$, $\lambda_{enc} = -2\pi R\sigma$ and so

$$E(\rho) = \frac{-2\pi R\sigma}{2\pi\varepsilon_0\rho} = \frac{-R\sigma}{\varepsilon_0\rho} \quad \text{for } r < \rho < R.$$

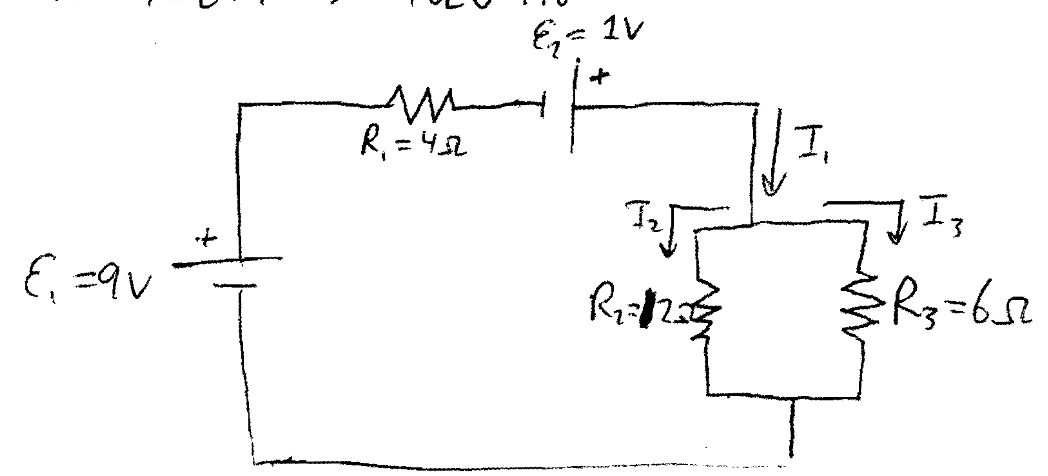
In summary: $\vec{E} = E(\rho)\hat{\rho}$ where

$$E(\rho) = \begin{cases} 0 & \rho < r \\ \frac{-R\sigma}{\varepsilon_0\rho} & r < \rho < R \\ 0 & \rho > R \end{cases}$$



PHYS 8B LECTURE 2 MIDTERM 1

PROBLEM 3 SOLUTION:

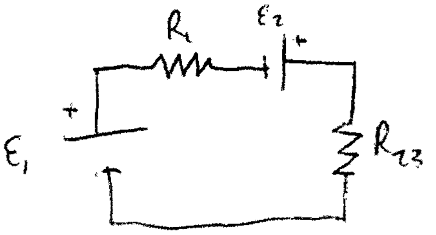


WE WANT POWER DISSIPATED BY 6Ω RESISTOR, WHICH I HAVE LABELLED R_3. THIS WE ~~K~~NOW TO BE

$P = I_3V_3 = I_3^2R_3 = \frac{V_3^2}{R_3}$. R_3 is given, so WE NEED TO FIND I_3 or V_3. THERE ARE TWO POSSIBLE SOLUTIONS HERE:

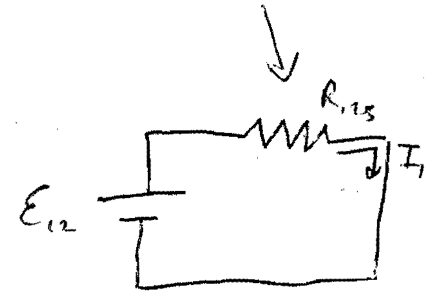
(A) EQUIVALENT RESISTANCES:

we will reduce our circuit:



$$R_{23} = \frac{R_2R_3}{R_2+R_3} = 4\Omega$$

b/c in PARALLEL



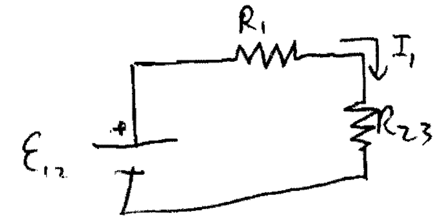
$R_{123} = 4\Omega + 4\Omega = 8\Omega$ b/c in SERIES

$\mathcal{E}_{12} = 9V + 1V = 10V$ b/c in SERIES

so we get $I_1 = \frac{\mathcal{E}_{12}}{R_{123}} = \frac{10V}{8\Omega} = \frac{5}{4}A$

NOW WE FIND THE VOLTAGE ACROSS R_3.

$V_2 = V_3$ because in PARALLEL



$$V_2 = V_3 = I_1 R_{23} = \left(\tfrac{5}{4}A\right)(4\Omega) = 5V$$

hence $$P_3 = \frac{V_3^2}{R_3} = \frac{(5V)^2}{(6\Omega)} = \boxed{\frac{25}{6}W}$$

(B) KIRCHOFF'S LAWS

SAME DIAGRAM AS IN (A)

PICK TWO LOOPS & ONE NODE

(1) $I_1 - I_2 - I_3 = 0$

(2) $\mathcal{E}_1 - I_1 R_1 + \mathcal{E}_2 - I_3 R_3 = 0$

(3) $I_2 R_2 - I_3 R_3 = 0$

(3) $\rightarrow$ $I_2 = I_3 \dfrac{R_3}{R_2}$

Applying (1) to (2)

$$\cancel{I_1 = I_3\left(1+\frac{R_3}{R_2}\right) = 0}$$

$$\mathcal{E}_1 - (I_2 + I_3)R_1 + \mathcal{E}_2 - I_3 R_3 = 0$$

$$\mathcal{E}_{12} - \left(I_3 \frac{R_3}{R_2} + I_3\right)R_1 - I_3 R_3 = 0$$

$$\mathcal{E}_{12} - I_3\left(\frac{R_3}{R_2}R_1 + R_1 + R_3\right) = 0$$

so $$I_3 = \frac{\mathcal{E}_{12}}{\left(\frac{R_3}{R_2}R_1 + R_1 + R_3\right)} = \frac{10V}{\left(\left(\frac{6\Omega}{12\Omega}\right)4\Omega + 4\Omega + 6\Omega\right)} = \frac{10V}{(2\Omega + 4\Omega + 6\Omega)} = \frac{5}{6}A$$

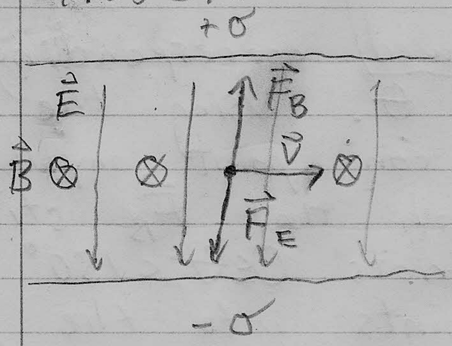
So $P_3 = I_3 V_3 = I_3^2 R_3$

$= \left(\frac{5}{6}A\right)^2 (6\Omega)$

$= \boxed{\frac{25}{6}W}$

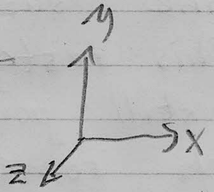
SAME RESULT

Problem 4



Uniform electric field downwards,

$$E = \frac{\sigma}{\varepsilon_0}.$$



Electric force must balance magnetic force for ions to move in a straight line and make it through the slit.

$$F_E = F_B \qquad F_E = qE = q\frac{\sigma}{\varepsilon_0}$$

$$F_B = qvB\sin\theta$$

$$\Rightarrow q\frac{\sigma}{\varepsilon_0} = qvB\sin\theta$$

$$\Rightarrow B = \frac{\sigma}{\varepsilon_0 v \sin\theta}$$

θ is angle between $\hat{v}$ and $\vec{B}$.

Minimum occurs when $\theta = 90°$.

$$\Rightarrow \boxed{B_{min} = \frac{\sigma}{\varepsilon_0 v}}$$

Use right-hand rule to figure out direction.

Thumb - $\vec{F}_B$ - up
Index - $\hat{v}$ - right
Middle - $\vec{B}$ - into page

$\vec{B}_{min}$ points $\boxed{\text{into page}}$ ($\otimes$) or

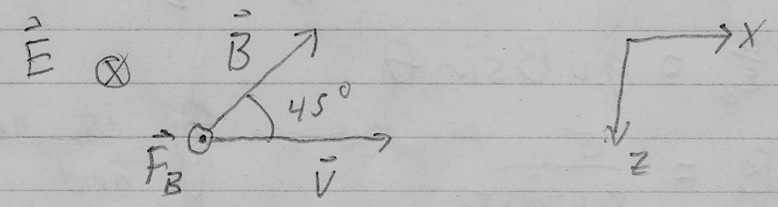
$$\vec{B}_{min} = -\frac{\sigma}{\varepsilon_0 v}\hat{z}.$$

Other values are allowed, corresponding to different values of θ. Using the right hand rule, $\vec{B}$ can point anywhere in the xz plane such that $\vec{F}_B$ still points up. This restricts $0 < \theta < 180°$ so that $\vec{B}$ still points into the page but with some x-component left or right.

Example: $\theta = 45°$, $\sin 45° = \frac{1}{\sqrt{2}}$

$$B = \frac{\sigma}{\varepsilon_0 V \sin 45°} = \frac{\sqrt{2}\,\sigma}{\varepsilon_0 V} > B_{min}$$

Top-down view:



$$\vec{B} = \frac{\sigma}{\varepsilon_0 V}\left(\hat{x} - \hat{z}\right)$$