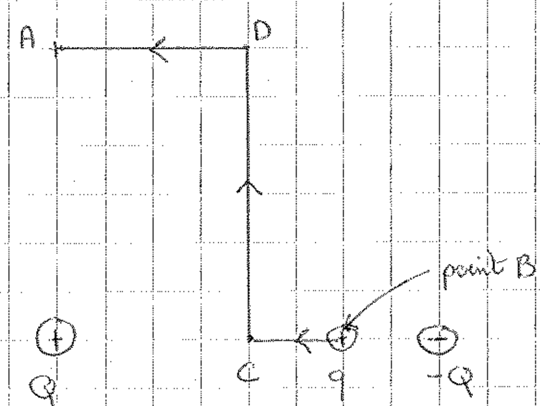
Physics 8B - Fall 2011 - Mid Term 1, October 5, 2011

Problem 1



<u>Key Idea:</u> <u>Static</u> electric fields are conservative.

$\boxed{\text{Thus the work done on the test charge is independent of the path!}}$

Be W the work done on the test charge, U_A, U_B the potential energy of the test charge at points A, B.

$W = + \Delta U = U_A - U_B = q(V_A - V_B)$ where V is the electric potential

By the principle of superposition, the electric potential at one point is the sum of the potentials created by each charge:

$V_B = V_B^+ + V_B^-$ where V_B^+ is the electric potential created by the charge $+Q$ at B.

Thus: $W = q\left(V_A^+ + V_A^- - V_B^+ - V_B^-\right)$

Now, points A and B are at the same distance of the charge $+Q$

hence $V_B^+ = V_A^+$

and $\boxed{W = q\left(V_A^- - V_B^-\right)}$

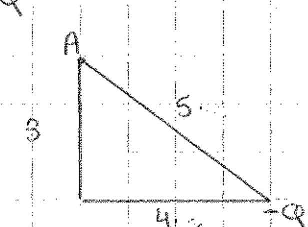
$\longrightarrow$ The work does not depend on the value of the positive charge.

We can actually consider that the positive charge is not there!

The electric potential created by a point charge $-Q$ is $\boxed{V = \dfrac{-Q}{4\pi\varepsilon_0 r} = \dfrac{-kQ}{r}}$

Thus $V_B^- = -\dfrac{kQ}{1} = -kQ$

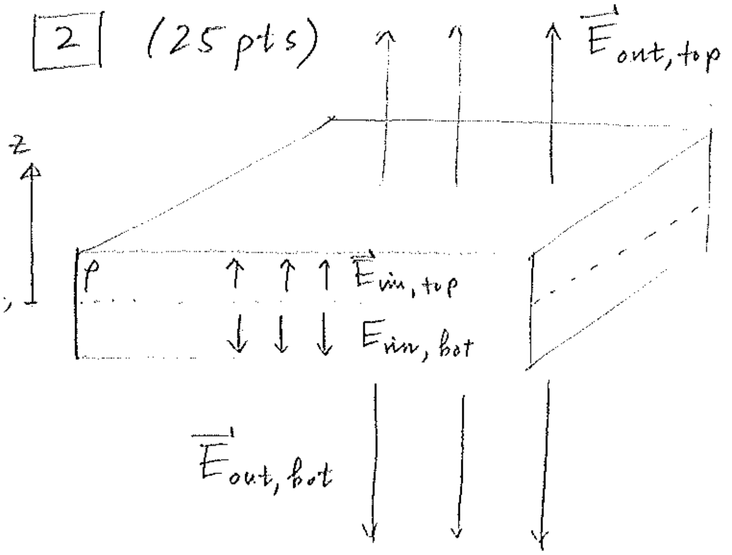
and $V_A^- = -\dfrac{kQ}{5}$ since:



The work done is:

$$\boxed{W = \frac{4}{5}\,kqQ} \qquad \left(k = \frac{1}{4\pi\varepsilon_0}\right)$$

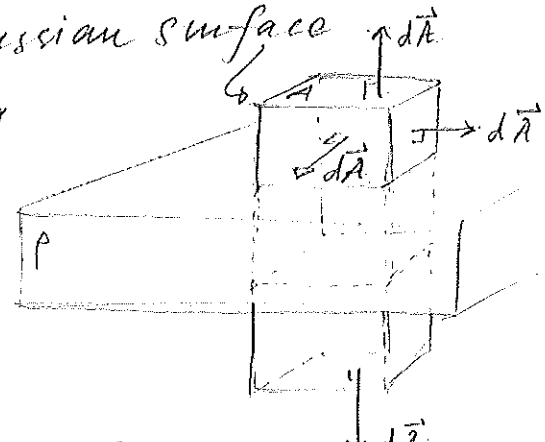
2 (25 pts)



By symmetry, we expect $\vec{E}$ to point in the $\hat{z}$ direction for $z>0$ and in the $-\hat{z}$ direction for $z<0$. We also expect $\vec{E}(z=0)=\vec{0}$, as well as $\vec{E}_{in,top} = -\vec{E}_{in,bot}$ and $\vec{E}_{out,top} = -\vec{E}_{out,bot}$.

Thus, we only need to find $|\vec{E}_{out}|$ and $|\vec{E}_{in}|$.

To find $|\vec{E}_{out}|$, consider the following Gaussian surface



Begin by writing Gauss' Law and evaluating the left-hand side and right-hand side separately.

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

integral over closed Gaussian surface

$$\oint \vec{E}\cdot d\vec{A} = \int_{top} \vec{E}\cdot d\vec{A} + \int_{bottom} \vec{E}\cdot d\vec{A} + \int_{left} \vec{E}\cdot d\vec{A} + \int_{right} \vec{E}\cdot d\vec{A} + \int_{front} \vec{E}\cdot d\vec{A} + \int_{back} \vec{E}\cdot d\vec{A}$$

$$= |\vec{E}_{out}|\left(\int_{top} dA \underbrace{\cos 0}_{=1} + \int_{bottom} dA \underbrace{\cos 0}_{=1} + \cancel{\int_{left} dA \underbrace{\cos\tfrac{\pi}{2}}_{=0}} + \cancel{\int_{right} dA \underbrace{\cos\tfrac{\pi}{2}}_{=0}} + \cancel{\int_{front} dA \underbrace{\cos\tfrac{\pi}{2}}_{=0}} + \cancel{\int_{back} dA \underbrace{\cos\tfrac{\pi}{2}}_{=0}} \right)$$

$$= |\vec{E}_{out}|(A + A) = 2A|\vec{E}_{out}|$$

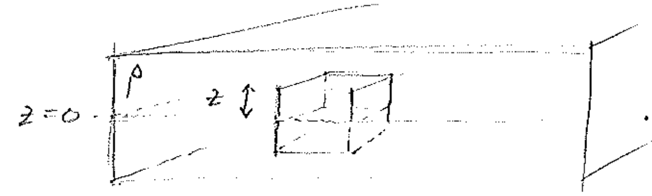
recall that $\rho = \frac{dQ}{dV}$:

$$\frac{Q_{encl}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int dQ = \frac{1}{\varepsilon_0}\int \rho\, dV = \frac{\rho}{\varepsilon_0}\int dV = \frac{\rho}{\varepsilon_0} A d$$

Thus, Gauss' Law gives: $2\cancel{A}|\vec{E}_{out}| = \frac{\rho}{\varepsilon_0}\cancel{A} d \Rightarrow \begin{cases} \boxed{\vec{E}_{out,top} = \frac{\rho d}{2\varepsilon_0}\hat{z}} \\ \boxed{\vec{E}_{out,bot} = -\frac{\rho d}{2\varepsilon_0}\hat{z}} \end{cases}$

Note that outside the slab, electric field is independent of z, as we have seen in an infinite sheet of charge.

To find $|\vec{E}_{in}|$, consider another Gaussian surface:



$$\oint \vec{E}\cdot d\vec{A} = \int_{top} \vec{E}\cdot d\vec{A} + \int_{bottom} \vec{E}\cdot d\vec{A} + \int_{left} \vec{E}\cdot d\vec{A}$$

$$+ \int_{right} \vec{E}\cdot d\vec{A} + \int_{front} \vec{E}\cdot d\vec{A} + \int_{back} \vec{E}\cdot d\vec{A}$$

$$= |\vec{E}_{in}| \Big(\int_{top} dA \underbrace{\cos 0}_{=1} + \int_{bottom} dA \underbrace{\cos 0}_{=1} + \int_{left} dA \underbrace{\cancel{\cos\tfrac{\pi}{2}}}_{=0} + \int_{right} dA \underbrace{\cancel{\cos\tfrac{\pi}{2}}}_{=0}$$

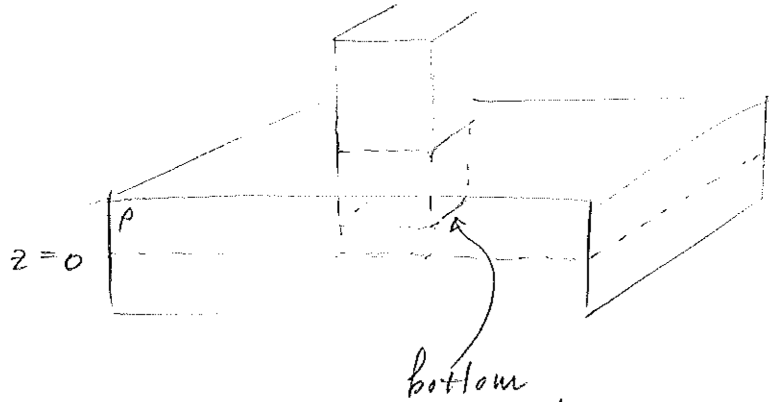
$$+ \int_{front} dA \underbrace{\cancel{\cos\tfrac{\pi}{2}}}_{=0} + \int_{back} dA \underbrace{\cancel{\cos\tfrac{\pi}{2}}}_{=0} \Big)$$

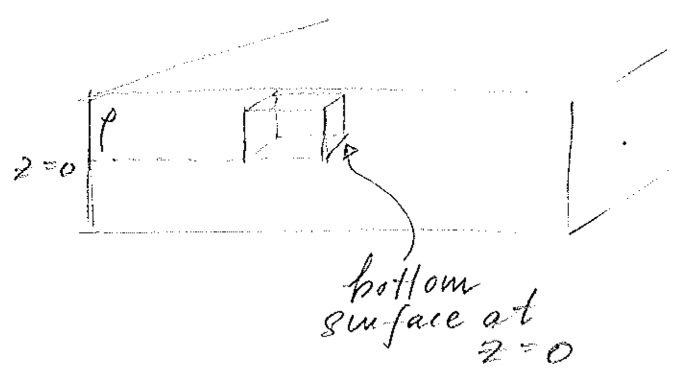
$$= |\vec{E}_{in}|(A + A) = 2A|\vec{E}_{in}|$$

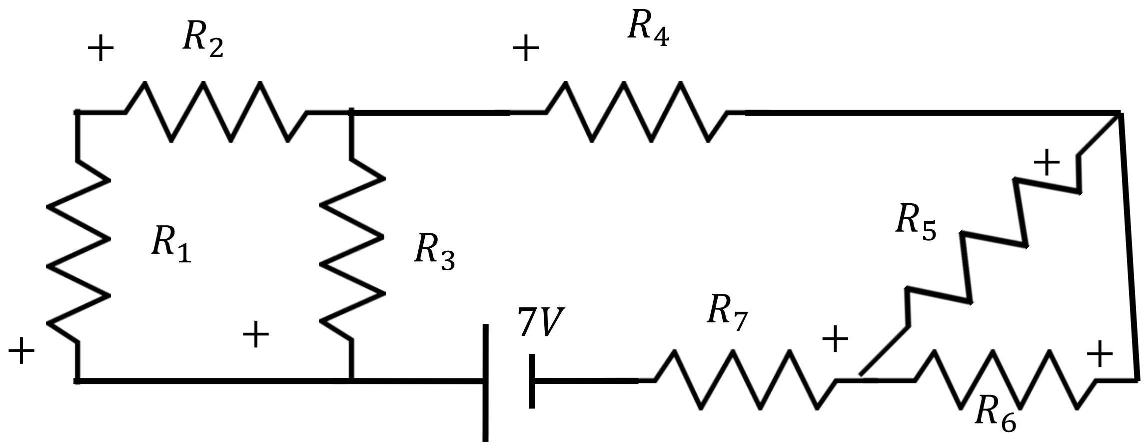
$$\frac{Q_{encl}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int dQ = \frac{\rho}{\varepsilon_0}\int dV = \frac{\rho}{\varepsilon_0} 2zA$$

$\therefore$ Gauss' Law yields $\cancel{2}\cancel{A}|\vec{E}_{in}| = \frac{\rho}{\varepsilon_0}\cancel{2}z\cancel{A} \Longrightarrow \begin{cases} \boxed{\vec{E}_{in,top} = \frac{\rho z}{\varepsilon_0}\hat{z}} \\ \boxed{\vec{E}_{in,bottom} = -\frac{\rho z}{\varepsilon_0}\hat{z}} \end{cases}$

Note that we could've also chosen the following Gaussian surfaces and gotten the same answers (try it!):







Higher potential is denoted by + on one end of each of the resistors

$$R_{12} = R_1 + R_2 = 12\Omega$$

$$R_{123} = \frac{1}{\frac{1}{R_{12}} + \frac{1}{R_3}} = \frac{1}{\frac{1}{12} + \frac{1}{6}}\Omega = 4\Omega$$

$$R_{56} = \frac{1}{\frac{1}{R_5} + \frac{1}{R_6}} = 5\Omega$$

$$R_{eq} = R_{123} + R_4 + R_5 + R_{56} + R_7 = 14\Omega$$

$$V = I_{total}R_{eq} \;\; \rightarrow \;\; 7V = I_{total} \times 14\Omega \; \rightarrow \; I_{total} = \frac{1}{2}A$$

$$V_4 = I_{total} \times R_4 = \frac{1}{2}V$$

$$V_7 = I_{total} \times R_7 = 2V$$

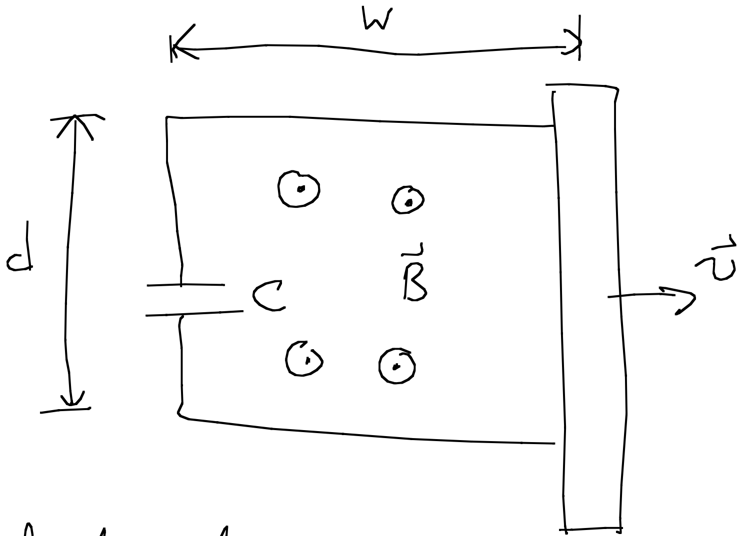
$$V_5 = V_6 = \frac{I_{total}}{2} \times R_5 = \frac{5}{2}V$$

$$V_3 = V_1 + V_2 = 7 - V_4 - V_5 - V_7 = 2V$$

$$I_1 = I_2 \; and \;\; R_1 = R_2 \;\; \rightarrow \;\; V_1 = V_2 = \frac{1}{2}V_3 = 1V$$

$$\boxed{V_1 = V_2 = 1V, \;\; V_3 = 2V, \;\; V_4 = \frac{1}{2}V, \;\; V_5 = V_6 = \frac{5}{2}V, \;\; V_7 = 2V}$$

Problem 4 solution (Phys 8B Section1 exam)



Induced EMF:

$|\mathcal{E}| = \left|\frac{d\phi_B}{dt}\right|$ so we need to solve for flux and differentiate it.

$$\left.\begin{array}{l} w = w_0 + vt \\ d = \text{const} \end{array}\right\} A = wd$$

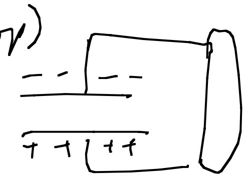
$$B = \text{const}$$

$$\phi_B = BA = Bwd = Bd(w_0 + vt)$$

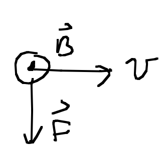
$$\left|\frac{d\phi_B}{dt}\right| = Bdv = |\mathcal{E}|$$

For a capacitor, $Q = CV$, and since there is no resistance, it is charged instantly (- on top)

$$\boxed{Q = CBvd}$$



To find the sign of charge on each plate, either use Lenz's law, or $\vec{F} = q(\vec{v} \times \vec{B})$

⊕ charge: 

⊖ charge;