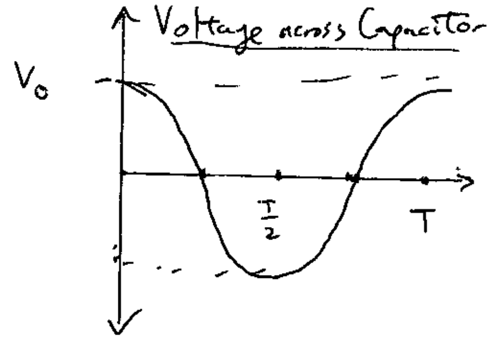
PROBLEM # 1

Lecture 2 (Thursday Exam)

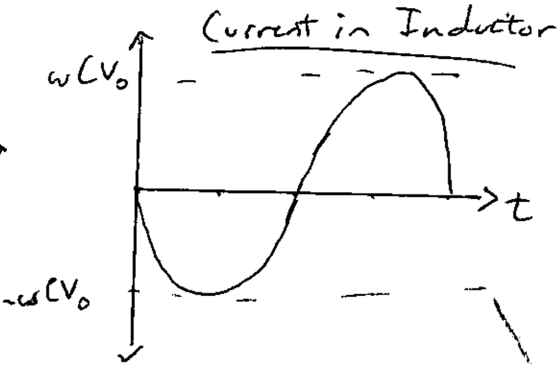
a) The capacitor is initially charged with $V(0) = V_0$. Thus $V(t) = V_0 \cos \omega t$



Since $I = \frac{dQ}{dt}$
$= C\frac{dV}{dt}$ $\Rightarrow$

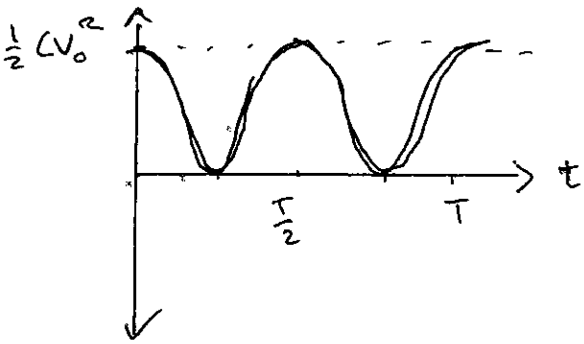
$I(t) = -\omega C V_0 \sin \omega t$

with $\omega = \frac{1}{\sqrt{LC}}$

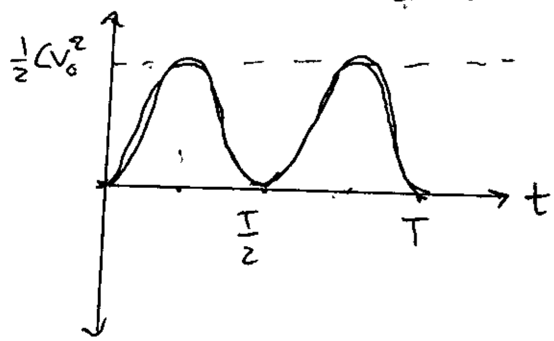


$\Downarrow$

Energy in Capacitor $= U_C = \frac{1}{2} C V^2$
$= \underbrace{\frac{1}{2} C V_0^2}_{U_{max}} \cos^2 \omega t$



Energy in Inductor $= U_L = \frac{1}{2} L I^2$
$= \frac{1}{2} C V_0^2 \sin^2 \omega t$



$$U_{max} = U_C(0) = U_L(T/4) = U_{tot}$$
$$= \frac{C V_0^2}{2} (\sin^2 \omega t + \cos^2 \omega t) = \boxed{\frac{1}{2} C V_0^2}$$

b) Here are two ways to do this:

i) Set $\frac{1}{2} C V^2 + \frac{1}{2} L I^2 = \frac{1}{2} C V_0^2 (= U_{tot})$

Then set $\frac{1}{2} L I^2 = \frac{1}{2} C V^2$ and solve:

$L I^2 = \frac{1}{2} C V_0^2 \Rightarrow \boxed{I = \sqrt{\frac{C V_0^2}{2L}}}$

ii) Using results from above, set

$\frac{1}{2} C V_0^2 \cos^2 \omega t = \frac{1}{2} C V_0^2 \sin^2 \omega t = \frac{1}{2} L I^2$

Equality holds when $\sin^2 \omega t = \cos^2 \omega t$:

$\frac{C V_0^2}{4} = \frac{1}{2} L I^2 \Rightarrow I = \sqrt{\frac{C V_0^2}{2L}}$

c) Again, we can use the results from above and solve by inspection:

$\frac{C V_0^2}{2} \sin^2 \omega t = \frac{C V_0^2}{2} \cos^2 \omega t$

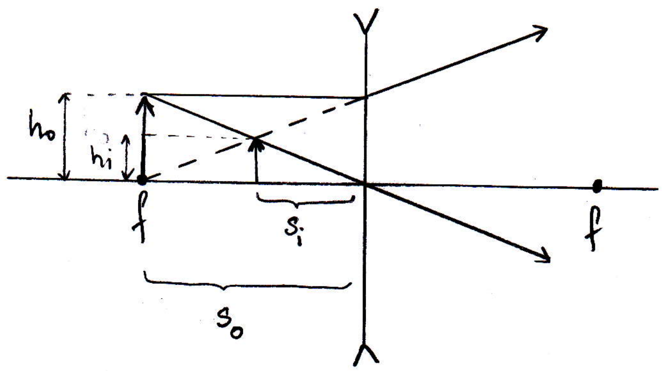
$\omega t = \frac{\pi}{4} \Rightarrow \boxed{t = \frac{\pi}{4} \sqrt{LC}}$ $\left(\begin{matrix} \text{as} \\ \omega = \frac{1}{\sqrt{LC}} \end{matrix}\right)$

Note the periodicity of the energy is half that of the voltage/current.

We could also use physical reasoning: If $U_C = U_L$, then both must be half the total energy. This occurs at $T/8$.

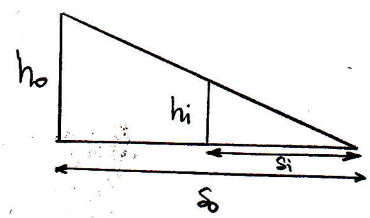
(2)

[Midterm 2 - Lecture 2.]



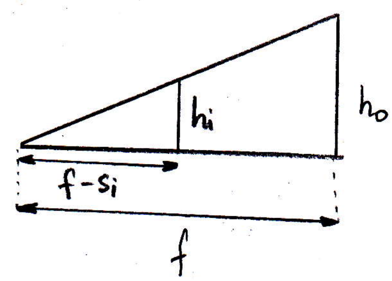
The image is virtual, upright, demagnified.

Using similar triangles:



$$\frac{h_i}{h_o} = \frac{|s_i|}{s_o}.$$

$$\frac{|f|-|s_i|}{|f|} = \frac{h_i}{h_o} = 1 - \frac{|s_i|}{|f|}.$$



$$\Rightarrow \quad 1 - \frac{|s_i|}{|f|} = \frac{|s_i|}{s_o}$$

$$\Rightarrow \quad \frac{1}{|s_i|} - \frac{1}{|f|} = \frac{1}{s_o}.$$ (divided by $|s_i|$).

$$\Rightarrow \quad \frac{1}{|f|} = \frac{1}{|s_i|} - \frac{1}{s_o}.$$

With $f < 0$ and $s_i < 0$ (diverging lens, virtual image):

$|f| = -f$, $|s_i| = -s_i$

$$\therefore \quad \frac{1}{|f|} = \frac{1}{|s_i|} - \frac{1}{s_o} \Rightarrow -\frac{1}{f} = -\frac{1}{s_i} - \frac{1}{s_o} \Rightarrow \boxed{\frac{1}{f} = \frac{1}{s_i} + \frac{1}{s_o}} \quad \blacksquare$$

Physics 8B - Section II - Exam 2

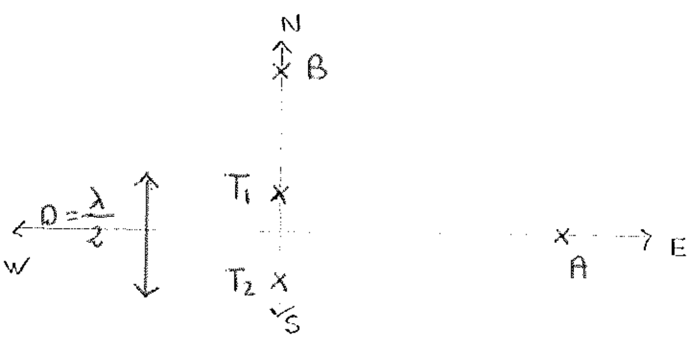
Problem 3

Key concepts: * "The power is very little in the East-West direction" means that the two radio waves are interfering destructively in this direction (EW).

* Similarly, in the North-South direction the power is big so that the interferences are constructive in the North-South direction.

* If $\Delta\varphi$ is the difference of phase between the two waves at one point, we have constructive interferences if $\Delta\varphi = 2m\pi$
destructive interferences if $\Delta\varphi = (2m+1)\pi$

Be T_1, T_2 the two towers, A a place in the East-West direction, B a place in the North-South direction:



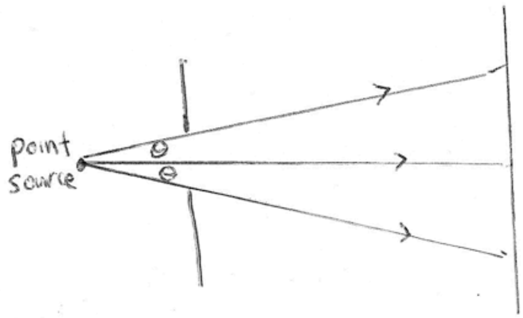
Suppose the two waves are initially emitted with a phase difference φ_0.
The difference of path length (DPL) at point A is $DPL|_A = 0$
at point B is $DPL|_B = \frac{\lambda}{2}$

Hence, the difference of phase at point A: $\Delta\varphi|_A = \varphi_0 + 2\pi \frac{DPL|_A}{\lambda} = \varphi_0$
at point B: $\Delta\varphi|_B = \varphi_0 + 2\pi \frac{DPL|_A}{\lambda} = \varphi_0 + \pi$

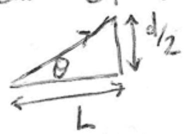
and we want $\begin{cases} \Delta\varphi|_A = (2m+1)\pi \\ \Delta\varphi|_B = 2m\pi \end{cases} \Longrightarrow \varphi_0 = (2m+1)\pi$: The signals have to be initially out of phase by half a period

Q4

(a) If we assume light behaves like a particle, then it will <u>not</u> undergo any diffraction. The light rays will travel in straight lines towards the screen as shown:



There will be a circle of light on the screen.
The "angular smear" of this circle can be calculated using trigonometry



$\tan\theta = \frac{d}{2L}$

So $\theta = \tan^{-1}\left(\frac{d}{2L}\right)$ ($\approx \frac{d}{2L}$ for small θ)

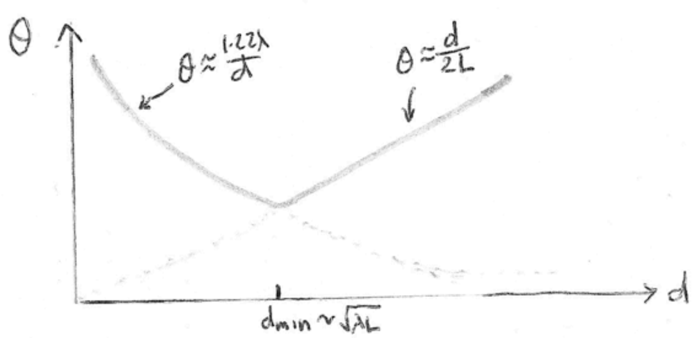
(b) If d decreases, then the angular smear decreases since $\theta = \tan^{-1}\left(\frac{d}{2L}\right)$

(c) We now assume the light has a wavelength λ, and consider diffraction effects. For $d \ll L$, the light will arrive at the slit approximately in phase and thus the standard formula for diffraction at a circular aperture will apply.
The angular smear will be approximately given by the first minimum:

$$\sin\theta = \frac{1.22\lambda}{d}$$

$$\theta = \sin^{-1}\left(\frac{1.22\lambda}{d}\right) \quad \left(\approx \frac{1.22\lambda}{d} \text{ for small } \theta\right).$$

The combination of parts (a) and (c) implies that the angular smear as a function of d is:



(d) From the graph we see that there is a minimum angular smear. This occurs when the "diffraction smear" from part (c) is of the same magnitude as the "particle smear" from part (a). That is, when

$$\frac{d}{2L} \approx \theta \approx \frac{1.22\lambda}{d}$$

$$d^2 \approx 2.44\lambda L$$

$$d \sim \sqrt{\lambda L}$$

So in order to obtain the minimum possible angular smear you should make the hole of size $d \sim \sqrt{\lambda L}$.

(Note: in deriving this, the small angle approximation is used, which will be consistent provided $\theta_{min} \approx \sqrt{\frac{\lambda}{L}} \ll 1$, that is, $\lambda \ll L$).