Solution to problem 1, exam 1.

a. $\Delta V_L + \Delta V_R = 0$ (loop rule)

$\Delta V_R = -I_0 R \Rightarrow \Delta V_L = +I_0 R.$

b. $U_L = \frac{1}{2} L I^2$. $U_L \to U_L/2 \Rightarrow I \to I/\sqrt{2}$.

Thus, $I = I_0/\sqrt{2}$.

c. Way 1:

$\Delta V_L + \Delta V_R = 0$ (loop rule)

$\Delta V_R = -I_0 R/\sqrt{2} \Rightarrow \Delta V_L = +I_0 R/\sqrt{2}$

Way 2:

$$\Delta V_L = -L \frac{dI}{dt}.$$

$$I = I_0 e^{-Rt/L} \Rightarrow \frac{dI}{dt} = -\frac{R}{L} I_0 e^{-Rt/L} = -\frac{RI}{L}$$

$$\Rightarrow \Delta V_L = RI. \quad I = I_0/\sqrt{2} \Rightarrow \Delta V_L = RI_0/\sqrt{2}.$$

Problem 2

(a) The electric field between the plates is given by: $E = \frac{\sigma}{\varepsilon_0}$.

+5 where $\sigma = \frac{Q}{A}$ is the charge per unit area on the plates.

Hence $\boxed{Q(t) = \varepsilon_0 A E(t)}$ is the charge on the plates as a function of electric field E between the plates.

(b) By definition: $\boxed{I = \frac{dQ}{dt}}$

+2

(c) From (a) and (b):

+4

$$I = \frac{dQ}{dt} = \frac{d}{dt}\left\{ \varepsilon_0 A E(t) \right\}$$

$$\boxed{I = \varepsilon_0 A \frac{dE}{dt}}$$

(d). Electric Flux: $\Phi_E = \int \vec{E} \cdot \vec{dA}$.

+4 For the entire plate, the total flux is $\Phi_E = EA$.

Then we can rewrite the current as:

$$I = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 \frac{d}{dt}(EA)$$

$$\boxed{I = \varepsilon_0 \frac{d\Phi_E}{dt}}.$$

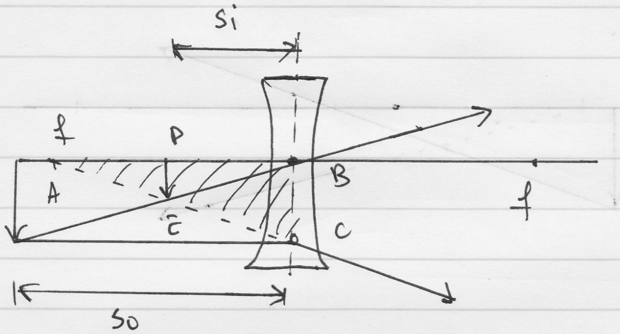
Since I is given to be non-zero, we can conclude that we have a nonzero change in electric flux. $\frac{d\Phi_E}{dt} \neq 0$.

(e) Yes, there will be.

+10 The (modified) Ampere's law : $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$.

Even if the Amperian loop chosen does not enclose any current, since the electric flux through said loop changes with time (i.e. $\frac{d\Phi_E}{dt} \neq 0$), there will be magnetic field created.

Problem 3



Since the lens diverges the image is virtual and demagnified. The ray tracing diagram is shown above. From the similar triangles ΔADE and ΔABC,

$$\frac{BC}{DE} = \frac{h_o}{h_i} = \frac{f}{f - s_i} .$$

The magnification of the lens implies $M = \dfrac{h_o}{h_i} = \dfrac{s_o}{s_i}$

$$\therefore \quad \frac{s_o}{s_i} = \frac{f}{f - s_i}$$

$$s_o f - s_o s_i = f s_i$$

$$s_o f - f s_i = s_o s_i$$

$$f = \frac{s_o s_i}{s_o - s_i}$$

$$\therefore \quad \frac{1}{f} = \frac{s_o - s_i}{s_o s_i} = \frac{1}{s_i} - \frac{1}{s_o}$$

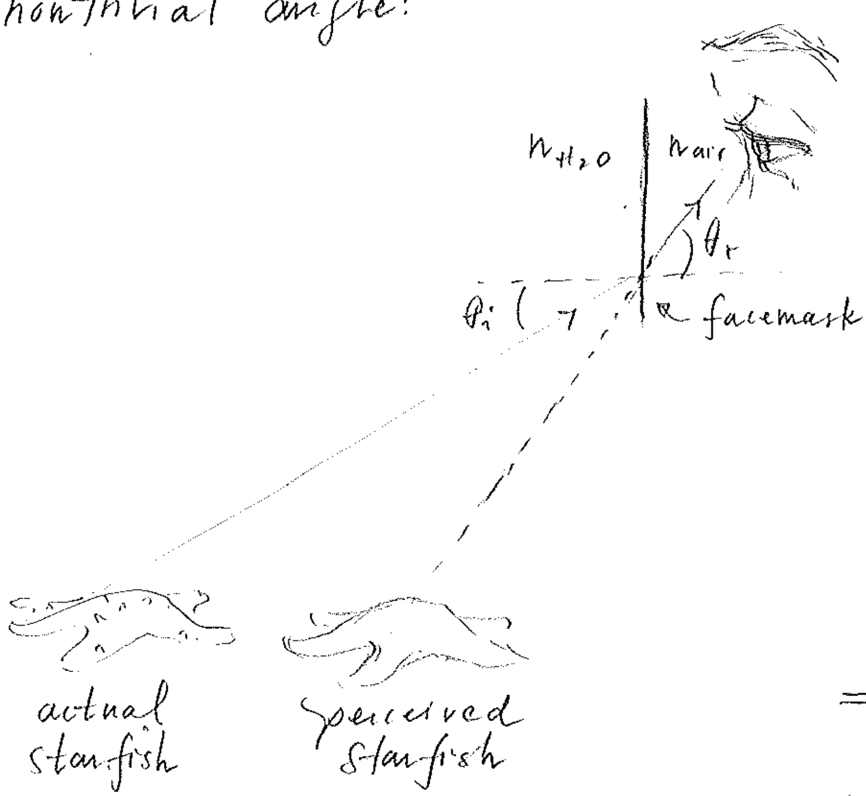
$$\boxed{\frac{1}{(-f)} = \frac{1}{s_o} + \frac{1}{(-s_i)}}$$

in agreement with the lens equation.

8B Midterm 2
Problem 4 solution

The facemask helps the diver to see the fish by providing a pocket of air around the diver's eyes. Human eyes have evolved such that they depend upon the refraction at the air-cornea interface to correctly form an image on the retina. The index of refraction of the cornea/vitreous humour is approximately equal to that of water. Thus, light entering the eye from water does not refract sufficiently to form a clear image on the retina.

The diver sees the starfish at an angle on the ocean floor that is closer to him than the actual location of the starfish. This is because the diver has shifted his gaze and not his head, so rays from the starfish are now incident upon the plane of the facemask at a nontrivial angle:



Snell's Law:

$$n_{H_2O} \sin\theta_i = n_{air} \sin\theta_r$$

$$n_{H_2O} > n_{air}$$

$$\Rightarrow \theta_i < \theta_r$$