Practice Midterm 1

1. When asked for speed in this type of problem, immediately think of energy conservation ($\Delta KE = \Delta PE$). In this case, the charges start with a triangular setup where they have potential energy, since they have the same charge they naturally repel one another and move away from one another once released. At very far away, the electric force becomes practically 0, so PE = 0, and all potential energy has converted into kinetic energy.

The next step is to figure out the potential energy. You know that for 1 charge q: PE = U = qV, where V is the potential caused by the other charges at the position of q. Since we have 3 identical charges of equal distances from each other, just find V that one of the charges feel (due to the other two), find its potential energy, multiply by 3 and divide by 2 (to avoid double counting). You know q and r (= L), so $V = kq/r$.

Sum this up in equations: (the kinetic energy is tripled because it's the sum of KE of 3 particles)

$$\frac{3}{2}mv^2 = KE = U = \sum qV = \frac{3}{2}\, qV_{tot} = \frac{3}{2}\, q(2)\left(\frac{kq}{r}\right) = 3\frac{kq^2}{L}$$

$$v = \sqrt{2\frac{kq^2}{mL}}$$

2. When asked for electric field, if you don't see a point charge, the first thing to try is Gauss's law.

a) Because Gauss's law concerns only enclosed charge, E outside a charged sphere is exactly like E of a point charge. All you need to do in part (a) is to find what the charge is, using $q = \rho V$.

$$E = \frac{kq}{r^2} = \frac{k}{r^2}\,(\rho V) = \frac{k}{r^2}\,\rho\left(\frac{4}{3}\pi R^3\right) = \frac{4\pi k}{3}\frac{\rho R^3}{r^2}$$

b) Again, a speed question most likely suggests conservation of energy. Find the change in potential energy of the particle from d to 2d, and equate it with $\frac{1}{2}\, mv^2$.

$$\frac{1}{2}mv^2 = \Delta U = Q\Delta V = Q\left(\int_{R+d}^{R+2d} E\, dr\right)$$

$$= Q\int_{R+d}^{R+2d} \frac{4\pi k}{3}\rho R^3\frac{dr}{r^2} = \frac{4\pi k}{3}\, Q\rho R^3\,\left(\frac{1}{R+d} - \frac{1}{R+2d}\right)$$

3. Remember that $E = \sigma/\epsilon_0$ between 2 plates of a capacitor. Both the proton and the electron feel this exact same electric field, hence the same electric force. If they were identical particles they would meet half way between the plates, but they have different masses, hence different accelerations. Recall that $x = \frac{1}{2}at^2$ is the distance travelled if the initial velocity is 0. We also know the sum of the distances (when the particles just pass each other) is L, the separation between 2 plates.

So we have 2 unknowns, x_p and x_e, and we need 2 equations:

$$x_p + x_e = L \quad (eq.1)$$

$$x_p = \frac{1}{2}\, a_p t^2, \quad x_e = \frac{1}{2}\, a_e t^2$$

$$\rightarrow \frac{x_p}{x_e} = \frac{a_p}{a_e} = \frac{\left(\frac{F}{m_p}\right)}{\left(\frac{F}{m_e}\right)} = \frac{m_e}{m_p} \qquad (eq. 2)$$

Solve for x_p, since the question asks for the distance from the positive plate.

$$L = x_p + \frac{m_p}{m_e}\, x_p = x_p\left(1 + \frac{m_p}{m_e}\right) \quad \rightarrow \quad x_p = L\frac{m_e}{m_e + m_p}$$

4. Work = potential energy in this case. So the problem is to find the potential energy in this triangular configuration, exactly like in problem 1.

$$W = 3kQ^2/a$$

5. Not a point charge → use Gauss's law.
We need to find E before worrying about A.
Since the point of interest is in the shell, the enclosed charge is the sum of the charge Q at the center and part of the charge in the shell. First we find what that partial shell charge is:

$$q_{part} = \int_a^r \rho dV = \int_a^r \frac{A}{r}\, 4\pi r^2 dr = 2\pi A(r^2 - a^2)$$

The integral is from a to r since the total shell charge is from a to b and we're interested in the part enclosed inside the radius r.
Then Gauuss's law says:

$$E(4\pi r^2) = \frac{1}{\epsilon_0}\left(Q + q_{part}\right) = \frac{1}{\epsilon_0}\left(Q + 2\pi A(r^2 - a^2)\right)$$

$$E = \frac{1}{\epsilon_0}\left(Q + 2\pi A(r^2 - a^2)\right)\left(\frac{1}{4\pi r^2}\right) = \frac{1}{\epsilon_0}\left(\frac{A}{2} + \frac{1}{4\pi r^2}(Q - 2\pi A a^2)\right)$$

Here we separate the term independent of r and those dependent of r. The objective is to get a uniform E, i.e. E does not depend on r, so the term that does depend on r must vanish. Hence:

$$Q - 2\pi A a^2 = 0 \quad \rightarrow \quad A = \frac{Q}{2\pi a^2}$$

6.

<u>Finding E</u>:
Gauss's law won't work because there's no simple Gaussian surface whose area we know that can enclose the ring and give a uniform electric field everywhere on the surface.
So we need Coulomb's law in integral form (Note: $E = kq/r^2$ is true only for point charge and outside a sphere).

$$dE = k\frac{dq}{r^2} = k\frac{\lambda R d\theta}{R^2}$$

$dq = \lambda R\, d\theta$ since $Rd\theta$ is the arc length of the infinitesimal section of charge on the ring. For the denominator: $r = R$ because the distance between any point on the ring to the center is the radius

R. Recall that E is a vector, so each dE is also a vector, and all dE points in different directions toward the center. That means for every dE there is one other dE with exactly the same magnitude and opposite direction to cancel its effect. So $E_{tot} = 0$. (You didn't have to through the first equation with dq to reach this conclusion, I'm doing it here just in case we're given an incomplete circle, then not everything will cancel out.)

<u>Finding B</u>:

When the ring rotates, the charges on the ring also move with respect to the stationary viewer, so we have a current. Because this is not a straight wire, we can't use Ampere's law (there's no loop to enclose a circular current), and have to use Bio-Savart law instead:

$$d\boldsymbol{B} = \frac{\mu_0}{4\pi} \, I d\boldsymbol{l} \times \hat{\boldsymbol{r}} / r^2$$

Again, $r^2 = R^2$. Since the line element (dl) and the vector $\hat{r}$ are perpendicular to each other, the cross product gives dB in the upward direction (use Right Hand Rule), and:

$$I \, dl = \frac{dq}{dt} \, dl = dq \frac{dl}{dt} = dq \; v$$

$$\rightarrow \quad Idl = (\lambda R \, d\theta)(\omega R)$$

$$dB = \frac{\mu_0}{4\pi} \frac{\lambda R^2}{R^2} \, \omega \, d\theta \quad \rightarrow \quad B = \int_0^{2\pi} \frac{\mu_0}{4\pi} \frac{\lambda R^2}{R^2} \, \omega \, d\theta = \frac{\mu_0}{2} \, \lambda\omega$$

And B points upward.