RC Circuit and Kirchhoff's Law

<u>RC Circuit</u>: (a circuit containing both resistors and capacitors)

1. For a circuit with voltage source (e.g. battery), capacitors, and resistors:

- If the capacitor(s) starts out uncharged, i.e. Q=0, then since Q = VC, the voltage across the capacitor must also be 0. The capacitor temporarily acts like a wire (no resistance). To solve for current/voltage in the circuit: replace the capacitor(s) by a wire and simplify the circuit (now with only R's) by series and parallel rules.

- If the capacitor is fully charged, i.e. Q reaches its maximum value (Q = VC), then Q cannot be changing with time. Thus, $I = dQ/dt = 0$, i.e. no current flows through the capacitor, so the capacitor acts like a break in the circuit. To solve for current/voltage in the circuit: simply ignore the branches with C's, simplify the circuit (now with only R's) by series and parallel rules.

2. For a circuit with no voltage source:

- The capacitor (must be fully charged) acts like a battery whose voltage decreases with time.
- The equation is:

$$V_C - V_R = 0$$

$$V_C = \frac{Q(t)}{C}, \;\; V_R = I(t)R = R\frac{dQ(t)}{dt}$$

$$\frac{Q}{C} - R\frac{dQ}{dt} = 0 \;\rightarrow\; \frac{dQ}{dt} = \frac{1}{RC}Q \;\rightarrow\; \frac{dQ}{Q} = \frac{1}{RC}\,dt$$

$$\int_{Q_0}^{Q}\frac{dQ}{Q} = \int_0^t \frac{1}{RC}\,dt$$

Notice that $Q_0 > Q$, since at the beginning (t=0), the capacitor is fully charged, and it slowly discharges as time increases. Hence:

$$\ln(Q/Q_0) = -\frac{1}{RC}\,t < 0 \;\rightarrow\; Q = Q_0 e^{-t/RC}$$

(Q decreases with time)
So:

$$I = \frac{dQ}{dt} = \frac{Q_0}{RC}\,e^{-t/RC} = \frac{V_0}{R}\,e^{-t/RC} = I_0\,e^{-t/RC}$$

$$V_{capacitor} = \frac{Q}{C} = V_0 e^{-t/RC} \qquad \left(V_0 = \frac{Q_0}{C}\right)$$

(both I and V of capacitor decreases with time)
You can apply this differential equation solving procedure to any circuit, just remember to add the appropriate V_{batt} and use Kirchhoff's Laws for the right loop.

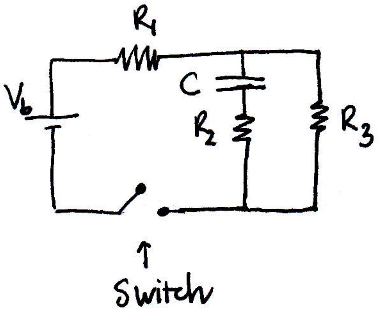
Kirchhoff's Laws: (to solve complicated circuits where you can't reduce the R's or C's to series or parallel groups, or when you have 2 or more batteries not directly connected in series with each other)

Decompose your circuits into loops. This means following the wire to make a complete circle, to the side redraw each loop with anything on that path and ignore everything else.

Pick a direction for the current in the loop. If the result gives a negative number, it just means the current is in the opposite direction of what you pick.

1. Kirchhoff's Voltage Law: pick a loop and a starting point with voltage V, follow the current, subtract after each drop in voltage, add if the current goes in the opposite direction or if there's a second battery connected in series.
2. Kirchhoff's Current Law: the sum of incoming currents minus the sum of outgoing currents equal 0.

The combination of these two laws on the loops gives you a set of equations, usually more than the number of unknowns, which guarantees that you can find the unknowns after a bit of substitution and algebra.



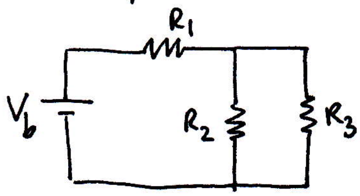
At first C is not charged.

* Close the switch ↔ turn on the supply and we have a complete circuit.

- Right after the switch is closed:

  C = wire

  ⇒ equivalent circuit:



(a)

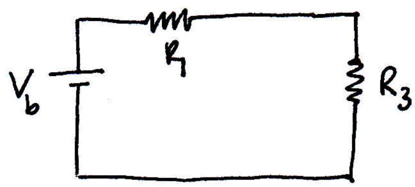
$((R_2 // R_3) + R_1)$

↑ "in series with"

- A long time after the switch is closed:

  C = break in circuit

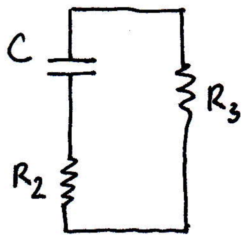
  ⇒ equivalent circuit:



(b)

$(R_1 + R_3)$

Now C is fully charged. Open the switch: equivalent circuit is:



(c)

$(R_2 + R_3)$

Follow the derivation on page 1, we have

$$I = \frac{Q_0}{(R_2 + R_3) C} e^{-t/((R_2+R_3)C)}$$

$$V_c = \frac{Q_0}{C} e^{-t/((R_2+R_3)C)}$$

where $Q_0 = V_3 C = I R_3 C = \left(\frac{V_b}{R_1 + R_3}\right) R_3 C$

↑ current through circuit (b).