Summary on Chapter 20 and 21:

1. **Coulomb's law**:
- (Force) = (field) * (charge): $\mathbf{F} = \mathbf{E}q$

Thus, when the charge q is positive, the force on it and the electric field on it are in the same direction, and when q is negative, the force and the electric field are in opposite directions.

- For point charge:

$$F = \frac{kQq}{r^2}, \quad E = \frac{kQ}{r^2}$$

In components:

$$E = \frac{kQ}{r^2}(\cos\theta \ \hat{x} + \sin\theta \ \hat{y})$$

- For a charge configuration:
--- if the configuration is a group of point charges, find E due to each point charge, then add the E vectors (by adding components)
--- if the configuration has a continuous distribution of charge, use:

$$E = k\int \frac{dq}{r^2} \ \hat{r}$$

------ Things to do when solving a problem with a charge configuration:
--------- determine the direction of E (which component of E would be cancelled?) by looking at the picture, if the x-component survives, multiply the integral by $\cos\theta$, if the y-component survives, multiply the integral by $\sin\theta$.
--------- how does E vary (with respect to angle, length, etc.)?
--------- rewrite dq in terms of density and the variable on which E is dependent if possible, a few examples are as follows:

	Density	Example	
1D	$\lambda = Q/L$	Straight wire	$dq = \lambda dr$
		Wire bent into a section of a circle with fixed radius	$dq = \lambda r \, d\theta$
2D	$\sigma = Q/A$	Disc	$dq = 2\pi\sigma r \, dr$
3D	$\rho = Q/V$	Ball of charge	$dq = 4\pi\rho r^2 \, dr$

--------- rewrite $\sin\theta$ and $\cos\theta$ in terms of r if possible
--------- integrate

2. **Electric dipole**:
---- dipole moment: $p = qd$, where $\pm q$ is the charge at each end of the dipole, d is the length of the dipole
---- net force on dipole: find the force on each end, sum them up. In a uniform electric field, the net force is 0.
---- torque: $\tau = p \times E = pE\sin\theta$
---- energy: $W = \int \tau \, d\theta = -pE\cos\theta$, evaluate from initial angle to final angle

3. **Gauss's Law**:
Flux through a surface:

$$\Phi = \int E \cdot dA = \int E \, dA \cos \theta$$

where θ is the angle between the electric field vector and the normal vector of the surface
When the magnitude of E is the same at every point on the surface,

$$\Phi = EA$$

Flux through a <u>closed</u> surface:

$$\Phi_{net} = \oint E \cdot dA = \frac{Q_{enclosed}}{\epsilon_0}$$

A closed surface means a surface containing a 3D volume, such as a sphere or a rectangular box. The integral above is over the Gaussian surface, which is an imaginary closed surface that we use to find the flux and the electric field at a certain point in space.
--- $Q_{enclosed}$ is the charge sitting inside that Gaussian surface.

$$\epsilon_0 = \frac{1}{4\pi k}$$

For symmetric configuration, i.e. the electric field is the same at every point on the surface, we have

$$E = \frac{Q_{enclosed}}{\epsilon_0 A}$$

1D	Straight wire, cylinder	$E = \dfrac{\lambda}{2\pi\epsilon_0 r}$
2D	Plane	$E = \sigma/\epsilon_0$
3D	Inside a sphere with uniform charge density	$E = \dfrac{\rho r}{3\epsilon_0}$
	Outside a sphere	$E = \dfrac{Q}{4\pi\epsilon_0 r^2}$

$E = 0$ inside a conductor (in electrostatic equilibrium). This means the space between two surfaces of a closed conductor does not have any electric field. All the charges in a conductor reside on its surfaces.

Examples 21.1, 21.3, 21.5, 21.6 in the textbook are useful.