Summary on Electromagnetism

Electricity:

Electric field acts parallel to the motion of charges. That means a charge would follow the electric field lines created by another charge to move toward or away from it.

Gauss' Law is your best friend. This should be the first thing to try when you're asked to calculate **electric field** of any charge configuration other than point charges.

Assume you are given a shape of charge and ask for the field at a point P a distance r away from the origin of that shape. Procedure:

1. Draw a "Gaussian surface" such that (a) the E field lines are perpendicular to the surface, **and** (b) the surface goes through the point P. Note: for a sphere: this surface encloses the whole sphere, but for infinitely long wire or infinitely large plane, this surface only encloses a section of the wire/plane.
2. Calculate the area of the surface; for the infinite plane, leave area as A because it will be cancelled.
3. Calculate the charge inside the surface. If the surface is inside the material: the size of the enclosed charge is the same as the size of the surface. If the surface is outside the material, the charge enclosed is the total charge.
4. $E = Q_{enc}/(\epsilon_0 A_{Gaussian\ Surface})$

If Gauss' law does not work (i.e. when the field is not constant everywhere on the surface, e.g. a ring of charge), use **Coulomb's law**: $E = k\int (dQ/r^2)$. Remember that E is a vector, so if some component of E gets cancelled due to symmetry of the charge configuration, then multiply the previous expression by either sin θ (if E_y survives) or cos θ(if E_x survives), then rewrite the sine or cosine in terms of y (or x) and r, and integrate. Most of the time, this results in the familiar $r^{3/2}$ factor in the denominator.

Electric Potential is a scalar, not a vector, so you don't have to worry about sine or cosine, everything sums up: $= k\int (dQ/r)$. If you can use Gauss' law to find E, do that, then integrate E over distance to find V: $|V| = \int E.dl$ (This can be seen in the following types of problem: find V in and outside two concentric shells, between a capacitor, between two lines of charge)

Electric Potential ENERGY: $U = qV$. If you encounter a question that has "velocity", think of 3 things right away: 1. kinetic energy $K = \frac{1}{2}mv^2$; 2. Conservation of energy; and 3. F=ma to relate with Coulomb force.

Capacitance: the ability to store charge at a given voltage.

Magnetism

Magnetic field acts perpendicularly to the motion of charge.

Magnetic force only presents when both of these conditions present: 1. Moving charge (i.e. current), 2. Magnetic field perpendicular to the motion of the charge (i.e. current direction)

- $\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B}$ for point charge (if there's an electric field, remember $\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$)
- $\boldsymbol{F} = I\boldsymbol{l} \times \boldsymbol{B}$ for current-carrying wire

RIGHT hand rule: *align your thumb with whatever is straight*. This means if you have a straight wire, then align your right thumb with the wire and the curl fingers show the direction of B, and if you have a wire loop, then curl your fingers with the direction of the current in the loop and the thumb is the direction of B.

When you deal with a loop of wire, if it's a:

1. Square loop: use $\boldsymbol{F} = I\boldsymbol{l} \times \boldsymbol{B}$ for each segment of the loop to find the force on each segment, then add up to find the net force, if asked. Note: the B field in this equation is *external* B, not the B field produced by the current in the wire. (This makes sense because things can't exert a force on itself)
2. Circular loop: use the same method as in finding E field for a loop of charge. Remember direction, as B is a vector: $\boldsymbol{B} = \frac{\mu_0}{4\pi} \int (Id\boldsymbol{l} \times \hat{\boldsymbol{r}}/r^2)$ (Biot-Savart law). If you are asked to find the B field of a circular disk (radius R) with some current density, break the disk into rings of radius r and integrate over r from 0 to R.

Ampere's law is analogous to Gauss' Law:

Assume you are given a current-carrying shape and ask for the B field at a point P a distance r away from the symmetry axis of that shape. Procedure:

1. Draw an "Amperian loop" such that (a) the B field lines are parallel to the loop, (b) the current goes ***through*** the loop, **and** (c) the loop goes through the point P.
2. Calculate the circumference of the loop. For infinite plane of current or infinitely long solenoid, just pick a rectangular loop of length *l* (the side parallel to the B field), *l* will cancel away: $B(2l) = \mu_0 nIl$
3. Calculate the current through the loop. If the loop is inside the material: the size of the enclosed current is the same as the size of the loop. If the loop is outside the material, the enclosed current is the total current.

$$B = \mu_0 I_{enc}/L_{Amperian\ loop}$$

Lenz's law: the induced current wants to **oppose** the **change** in magnetic **flux**. This means if the flux is increasing, the induced current is in such direction that it can produce a field opposite of the *external* field; if the flux is decreasing, the induced current produces a field in the same direction as the *external* field.

Flux: $\phi = \boldsymbol{B}.\boldsymbol{A} = BA\cos\theta$, where θ is the angle between the B vector and the A vector. Note: the A vector is defined to be *perpendicular* to the surface.

There are at least 3 ways for flux to change: changing B field, changing Area, changing θ.

Inductance: the ability to induce voltage in response to changing current.

Note: the magnetic flux through a solenoid is the flux through each loop multiplied by the total number of loops: $\phi = NBA$ (A: area of each loop)

Ideal transformer: the flux through each coil in inductor 1 is equal to the flux through each coil in inductor 2, i.e. $\phi_1 = \phi_2 = \phi$

$$V_1 = N_1\frac{d\phi}{dt}; \quad V_2 = N_2\frac{d\phi}{dt} \quad \rightarrow \frac{V_1}{N_1} = \frac{V_2}{N_2}$$

Circuits

Capacitance C (unit: Farad), Resistance R (unit: Ohm), Inductance L (unit: Henry) are intrinsic property of capacitor, resistor, and solenoid correspondingly, depends solely on the material, shape, size, etc. of the object.

$C = \epsilon A/d$ for plate capacitor only, $R = \rho l/A$

In general:

- To find C: 1. Find E (using Gauss' Law); 2. Find $V = \int E.dl$; 3. Use $C = Q/V$
- $R = V/I$
- $|V| = L\frac{dI}{dt} = N\frac{d\phi}{dt} = N\frac{d(BA)}{dt} = N\left(A\frac{dB}{dt} + B\frac{dA}{dt}\right).$
  If A is constant in time, and B is the field generated by the current in a solenoid, then:
  $$L\frac{dI}{dt} = NA\frac{dB}{dt} = NA\frac{d}{dt}(\mu_0 nI) = \mu_0 n^2 lA\frac{dI}{dt}$$
  So: $L = \mu_0 n^2 lA$, with n being the number of turns per unit length, l is the length of the solenoid.

Kirchhoff's laws: pick a direction for the current in the loop. If the result gives a negative number, it just means the current is in the opposite direction of what you pick.

1. Kirchhoff's Voltage Law: pick a loop and a starting point with voltage V, follow the current, subtract after each drop in voltage, add if the current goes in the opposite direction or if there's a second battery connected in series.

2. Kirchhoff’s Current Law: the sum of incoming currents minus the sum of outgoing currents equal 0.