

Worksheet T2 Solution

PART 1

Problem 1

You want the inner circumference of the ring to be equal to the outer circumference of the pipe.

$$2\pi r_r = 2\pi r_p$$

$$r_r = r_{r0} (1 + a_1(T_f - T_i)) = r_p = r_{p0} (1 + a_2(T_f - T_i))$$

Solve for T_f

$$T_f = \frac{r_{p0}(1 - a_2 T_i) - r_{r0}(1 - a_1 T_i)}{r_{r0} a_1 - r_{p0} a_2} = -75^\circ C$$

Problem 2

In this problem it's not clear to me whether a_1 is the linear expansion coefficient or the volume expansion coefficient (β), so for simplicity I will assume it to be the latter. The answer would change by a factor of 3 if you assume the former.

$$\Delta V = 0.01 V_0 = V_0 a_1 \Delta T$$

$$\Delta T = \frac{0.01}{a_1} = 5000^\circ C$$

PART 2

Problem 1

(a) For a distribution of speed $f(v)$, $N = \int_0^\infty f(v) dv$ is the total number of particles with speed between 0 and infinity. Thus, the number of particles with speed between v and dv is $N_{dv} = \int_v^{v+dv} f(v) dv$

(b) For finite distribution, the average of $g(v)$ is:

$$\langle g(v) \rangle = \frac{N_1 g(v_1) + N_2 g(v_2) + \dots + N_n g(v_n)}{N}$$

Where N is the sum of N_1, N_2, \dots, N_n , and each $g(v_i)$ is the function g evaluated at v_i . Change into integral form:

$$\langle g(v) \rangle = \frac{1}{\int_0^\infty f(v) dv} \left(\int_0^{v_1} g(v) f(v) dv + \int_{v_1}^{v_2} g(v) f(v) dv + \dots \right)$$

$$\langle g(v) \rangle = \frac{\int_0^\infty g(v) f(v) dv}{\int_0^\infty f(v) dv}$$

(c) Substitute 1 for $g(v)$

(d) Substitute $\frac{1}{2} m v^2$ for $g(v)$, you should get $\frac{3}{2} kT$ upon appropriate u -substitution for the integral, which agrees with the equipartition theorem (summary in worksheet T1).

Problem 2

$$(a) \langle v \rangle = v_{avg} = \frac{2*10+4*12+2*14+1*15+1*17}{10} = 12.8 \text{ (m/s)}$$

$$v_{rms} = \sqrt{\frac{1}{10} (2 * 10^2 + 4 * 12^2 + 2 * 14^2 + 15^2 + 17^2)} = 12.97 \text{ (m/s)}$$

$$(b) \frac{1}{2}mv_{rms}^2 = \frac{3}{2}NkT, N = 10$$

$$T = 5Nk/mv_{rms}^2$$

$$(c) E = 5/2 NkT$$

(d) The sample is too small

PART 3

Note:

In these problems, use common sense to know the direction of heat flow, set both the emitted heat and the absorbed heat positive and equal to each other. This means on the emitted heat side, you want $T_i - T_f$ instead of $T_f - T_i$.

If the specific heat and latent heat are given in unit of K, remember to convert all temperatures in the problem to K.

Problem 1

$$m_c C_{Cu}(T_c - T_f) = m_w C_w(T_f - T_w) + C_b(T_f - T_b)$$

Solve for T_f

$$T_f = \frac{m_c C_c T_c + m_w C_w T_w + C_b T_b}{m_c C_c + m_w C_w + C_b} = 293 K = 20^\circ C$$

Problem 2

Emitted heat: steam at 100C becomes water at 100C, then water at 100C cools down to 50C

Absorbed heat: ice at -10C heats up to ice at 0C, then becomes water at 0C, then water at 0C heats up to 50C

$$m_s L_v + m_s c_w(100 - 50) = m_i c_i(0 - (-10)) + m_i L_f + m_i c_w(50 - 0)$$

Solve for m_s

$$m_s = \frac{m_i(10c_i + L_f + 50c_w)}{L_v + 50c_w} = 164(g)$$