Worksheet T3 Solutions

PART 1

Question 3: Equal 100C (Use the equation for rate of heat flow to solve for T(midpoint) if you want to, you should get T1 + T2 = 2T(midpoint).)

Question 4: Heat flow is proportional to area, so with the area doubled, the rate heat flow is doubled (100J/s). At midpoint, T = 100C.

Question 5: T(midpoint) = 100C. Metal conducts better than wood, so the rate of heat flow must be greater than 50J/s.

Question 6:

(a) Think about heat as fluid flow, you don't want heat to build up at any point in the rod, or the temperature of the section where heat builds up would rise with time (thus violates the assumption that each point on the rod maintains constant temperature over time). So $H_{in} = H_{out}$

(b) $H_{wood} = \frac{50J}{s}$, and this rod has both wood and metal with constant H everywhere, so you can consider it as a mixed material with $k_{wood} < k < k_{metal}$. Hence, $H > \frac{50J}{s}$

(c) $H_{wood} = H_{metal}$

(d) $k_{metal} \frac{A}{l} (T_1 - T_m) = k_{wood} \frac{A}{l} \ (T_m - T_2)$

So: $k_{metal}(200 - T_m) = k_{wood} T_m$

$$(k_{metal} + k_{wood}) T_m = 200 k_{metal}$$

$$\left(1 + \frac{k_{wood}}{k_{metal}}\right) T_m = 200$$

Because $k_{wood} < k_{metal}$, $1 + k_{wood}/k_{metal} \ < 1 + 1 = 2$, therefore $T_m >$ 100C

Problem 1:

(a) $T_m = 200/(1 + \frac{k_{wood}}{k_{metal}})$. So $T_m = 187^o C$

(b) $H = k_w \frac{A}{l} \ (T_m - 0) \sim 0.1 J/s$

Problem 2:

We want the heat emitted by the water underneath the ice into the air above the ice to be equal to the heat required to transform water into ice.

$$Q_{emit} = m_{ice} L_f = \rho_{ice} A D L_f$$

Where D is the thickness of ice forming underneath the existing ice slab, and A is the surface area of the container. The question is asking for dD/dt, which we can get from:

$$\frac{dQ}{dt} = \ \rho_{ice} A L_f \frac{dD}{dt}$$

Since the density, area, and latent heat are constant with time. On the other hand, the rate of heat flow is given by:

$$\frac{dQ}{dt} = k_{ice} \frac{A}{D_0 + \ D} \ (T_{water} - \ T_{air})$$

But judging from the question which asks for the rate in cm/hour, we can safely assume that D is much smaller than D_0 and set $D_0 + D \sim D_0 = 5cm.$
Now we get:

$$\rho_{ice} A L_f \frac{dD}{dt} = k_{ice} \frac{A}{D_0} (T_{water} - T_{air})$$

$$\frac{dD}{dt} = \frac{k_{ice}(T_{water} - T_{air})}{\rho_{ice} L_f D_0}$$

$T_{water} = T_{ice} = 0^0 C$ because the water has to reach 0C to start forming ice. Remember to convert everything into SI units. Then $\frac{dD}{dt} = 1.1 * 10^{-6} m/s = 0.4 cm/hr$

PART 2

Question 6:

$P_1 = P_2$, if $\epsilon_1 = 4\epsilon_2$, then $T_1^4 = \frac{1}{4} T_2^4$, so $T_1 = T_2/\sqrt{2}$.

If $r_1 = 2r_2$, then $A_1 = 4A_2$, so $T_1 = T_2/\sqrt{2}$

Problem 1

(a) $P_{in} = P_{out}$. So:

$$\epsilon_{sun} \sigma A_{sun} T^4 = \epsilon_E A_{orbit} S$$

$\epsilon_{sun} = \epsilon_E = 1$ for perfect black bodies. $A_{sun} = 4\pi r_{sun}^2$, $A_{orbit} = 4\pi r_o^2$. Then:

$$S = \sigma \frac{r_s^2}{r_o^2} T^4$$

(b) S = 1360W/m^2

(c) The difference comes from the assumption that the earth and the sun are perfect black bodies.

(d) Assume that the earth emits all the heat it absorbs from the Sun, so P_in = P_out, i.e.:

$$\epsilon \pi r_e^2 S = \epsilon \sigma 4 \pi r_e^2 T_e^4$$

Notice the area on the left hand side is the cross sectional area which receives radiation from the sun, while the area on the right hand side is the surface area of the earth through which the earth's radiation is emitted.

$$S = 4\sigma T_e^4 = \sigma T_s^4 \left(\frac{r_s}{r_0}\right)^2$$

$$T_e = \frac{1}{\sqrt{2}} T_s \sqrt{\frac{r_s}{r_o}} \sim 278K = 5C$$

The actual temperature of the Earth is about 15C, but that takes into account the heat generated at the core and so on. We're only looking at the temperature that arises from the sun's radiation.