Note:

$$\Delta E = Q - W_{\boldsymbol{by}\ the\ gas} = Q + W_{\boldsymbol{on}\ the\ gas}$$

When a gas expands, it pushes things away, i.e. it does work on the environment, i.e. $W_{by\ the\ gas} > 0$ (i.e. $W_{on\ the\ gas} < 0$). When the volume of the gas decreases, that means something has to do work to compress it, i.e. $W_{by\ the\ gas} < 0$. Whichever W you choose to work with, you just need to be consistent with the sign and things will fall into place. On the exam, make sure you state clearly which convention you're using.

For a process that starts at (p_0, V_0) and ends at (p_1, V_1),

- If it's adiabatic, then $W = p_0 V_0^\gamma \left(\frac{V^{1-\gamma}}{1-\gamma}\right)_{V_0}^{V_1}$
- If it's isothermal, then $W = p_0 V_0 \ln\left(\frac{V_1}{V_0}\right)$

(Derivation was done in section on Thursday)

Solutions to the problems in T4

Problem 1

(a) $pV = NkT$. So $T_1 = p_0V_0/Nk$, $T_2 = 5p_0V_0/Nk$, $T_3 = T_1$ since the transformation 3-1 is isothermal.

(b) $p_2V_2^\gamma = p_3V_3^\gamma$ because 2-3 is adiabatic. $p_1V_1 = p_3V_3 = NkT_1$ because 3-1 is isothermal. Hence:

$$p_3 = p_1V_1/V_3$$
$$p_2V_2^\gamma = \frac{p_1V_1}{V_3} V_3^\gamma = p_1V_1\, V_3^{\gamma-1}$$
$$p_0(5V_0)^\gamma = p_0V_0V_3^{\gamma-1}$$
$$V_3 = \left(\frac{(5V_0)^\gamma}{V_0}\right)^{\frac{1}{\gamma-1}} = \mathbf{5^{\frac{\gamma}{\gamma-1}}} \boldsymbol{V_0}$$

(c) $p_3 = p_1V_1/V_3$

$$p_3 = \frac{p_0V_0}{5^{\frac{\gamma}{\gamma-1}}\,V_0} = \frac{\mathbf{1}}{\mathbf{5^{\gamma/(\gamma-1)}}}\,\boldsymbol{p_0}$$

(d) $\Delta E_{1-2} = E_2 - E_1 = \frac{5}{2}Nk(T_2 - T_1) = 10p_0V_0$

$\Delta E_{2-3} = -10p_0V_0$

$\Delta E_{3-1} = 0$. This is expected, since 3-1 is isothermal, no change in temperature means no change in internal energy.

(e) $W_{by\ the\ gas} = \int p dV$ is the area under the curve in pV diagram. For 1-2: p is constant, so work is just the area of the rectangle with sides p_0 and 4V_0, i.e. $W_{1-2} = 4p_0V_0$.

For 2-3: you can use the integral with gamma and all that jazz, or, since you already have the change in internal energy, knowing that Q=0 (because this part is adiabatic), $\Delta E_{2-3} = Q_{2-3} - W_{2-3} = -W_{2-3}$, you get $W_{2-3} = 10p_0V_0$.

For 3-1: do NOT think that the net work is 0 like the net change in internal energy. The diagram shows an area inside the loop, which means net work is nonzero. The arrows go clockwise, so the area is positive, i.e. the net work is positive. Anyway, use the integral formula to calculate work in this step.

$$W_{3-1} = \int_{V_3}^{V_1} p dV = p_3 V_3 \ln\left(\frac{V_1}{V_3}\right) = -\frac{\mathbf{7}}{\mathbf{2}} \mathbf{ln\, 5}\ \boldsymbol{p_0 V_0}$$

Notice that this is negative work done by the gas, since the gas is compressed.

(f) Use $Q = \Delta E + W$ to calculate heat for each portion.

Problem 2

(a) Step (i) is isothermal, hence $p_1 V_1 = p_2 V_2 = p_2(3V_1)$, so $p_2 = \frac{1}{3} p_1$

Step (iii) is adiabatic, hence $p_1 V_1^\gamma = p_3 V_3^\gamma$, so

$$p_3 = p_1 \left(\frac{V_1}{V_3}\right)^\gamma = \left(\frac{1}{3}\right)^\gamma p_1$$

(b) $T_1 = T_2$. Use $pV = NkT$ to find T's.

(c) Let us look at $\Delta E = Q - W$, for step (i), there is no change in T, hence $\Delta E = 0$, hence Q=W; since the gas is expanding, W>0, so Q>0, which means heat flows into the gas.

For step (ii), there is no change in V, hence W=0, hence $\Delta E = Q$. Point 2 has higher p than point 3, so it has higher T, so E must decrease from 2 to 3. Thus, Q is negative, i.e. heat flows out of the gas.

Step (iii) is adiabatic, so Q=0.

(d) Step (i): calculate the work done by the gas, $Q = W = \ln 3\ p_1 V_1$

Step (ii): calculate the change in E, where E= 3/2 NkT (monatomic). $Q = \Delta E = -\frac{3}{2} p_1 V_1 (1 - 3^{1-\gamma})$

(e) You can calculate the net work, but you can also look at the diagram, the arrows on the loop go clockwise, so the area inside the loop is positive. Therefore, the net work done by the gas is positive. (If the process starts from 1, then goes to 3, then goes to 2 and back to 1, then the arrows go counterclockwise and the net work is negative).

You can also say that the work done by the gas is greater than the work done on the gas, so the net work is positive in this case.

For net heat, again, $\Delta E = Q - W = 0$ (since internal energy goes back to its original value), W>0, thus Q>0

(f) This is a heat engine because it takes heat from the environment. A refrigerator would take heat ***out of*** the environment.

Problem 3

(a) Use $pV = NkT$ to calculate T, then use $E = \frac{3}{2} NkT$ (monatomic) to calculate ΔE.

Answer: $\Delta E_{AB} = 12 p_0 V_0$

(b) Work is the area under the curve, in this case it's the area under the upper curve, i.e. half of the ellipse plus the rectangle with sides 2p_0 and 4V_0.

$$W = \frac{1}{2} \pi p_0 (2V_0) + 2p_0(4V_0) = (8 + \pi) p_0 V_0$$

(c) $Q = W + \Delta E = (20 + \pi)p_0V_0$

(d) $\Delta E_{BA} = -\Delta E_{AB}$ since you want the net $\Delta E = 0$. W_{BA} is the area under the lower curve, and do NOT forget the MINUS sign, since this is work done **on** the gas, as the gas decreases in volume.

(e) Net work is the area inside the loop, and it's positive because the arrows go clockwise. $W_{net} = 2\pi p_0V_0$