PHYS 8B – Midterm 1
Review Session 4-5 PM
Saturday Oct 1, 2011 – 155 Dwinelle Hall
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Topics: Coulomb's law, Electric field, Dipole, Potential, Potential energy, Gauss's law, Capacitance, Dielectric, Ohm's law, Kirchoff's law

Tips on solving problems on the test:
- list the quantities *given* in the question
- list the equations that contain those given quantities: if more than one equations apply for the same quantity but in different situations (e.g., parallel vs. series circuits), consider which situation applies
- substitute
- ALWAYS write the equations with symbols and solve the questions in terms of symbols. Leave the numbers to the very end. NEVER plug in numbers while solving, that introduces only 3 problems: 1. Rounding errors or unit conversion errors, 2. Tracing the error is hard, and 3. Unhappy grader
- Remember UNITs

Three ways to find electric field E:
- Coulomb's law: $\boldsymbol{E} = k\int \left(\frac{dq\,\hat{\boldsymbol{r}}}{r^2}\right)$
- Gauss's law: $E = \frac{Q_{enc}}{\epsilon_0 A}$
- Derivative of potential: $E = \frac{dV}{dx}$ or $E = \Delta V/\Delta x$ for constant electric field (e.g., E inside a capacitor)

<u>Problem 1</u>:
A long cylinder of radius R carries a charge density that is proportional to the distance from the axis: $\rho = ks$, for some constant k.
(a) Find the electric field inside and outside the cylinder. (Hint: the cylindrical volume element is $dV = rdrd\phi dz$) [Griffiths, Ex. 2.3, p.72]
(b) Now a sphere of radius r is hollowed out of the cylinder (r < R). The center of the sphere lies on the axis of the cylinder. What is the electric field at d, where $d > R$?
(c) Find the work required to bring a charge Q from a distance d ($d > R$) to the surface of the cylinder (with the spherical hole inside).

Basic properties of conductors: [Griffiths, pp. 96-98]
- E = 0 inside the conducting material
- $\rho = 0$ inside the conducting material (no charge enclosed)
- Any net charge resides on the surface of the conducting material
- A conductor is an equipotential, regardless of its shape, i.e., V is constant everywhere on its surface.
- E is perpendicular to the surface, just outside the conducting material

<u>Problem 2</u>: [Griffiths, Prob. 2.36, p. 101]

Two spherical cavities of radii a and b are hollowed out from the interior of a neutral conducting sphere of radius R. At the center of each cavity is a point charge, let us call them q_a and q_b.

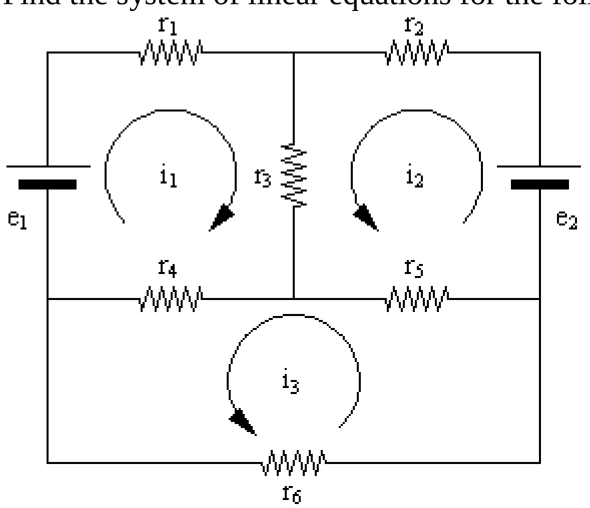
(a) Find the surface charges σ_a, σ_b and σ_R
(b) What is the field outside the conductor?
(c) What is the field inside each cavity?
(d) What is the force on q_a and q_b?
(e) Which of these answers would change if a third charge is placed outside the conductor?

Problem 3:
The space between the plates of a parallel-plate capacitor is filled with 2 slabs of linear dielectric material. The slabs have thicknesses a_1 and a_2 and dielectric constants κ_1 and κ_2. The free charge densities on the top and bottom plates are σ and $-\sigma$, respectively. The surface area of each plate is A.

(a) Find E in each slab
(b) Find the potential difference between the plates
(c) After the capacitor is fully charged, it is connected to a resistor R. Find the time it takes to discharge the capacitor by half.

Kirchoff's Law:
First, ALWAYS try to solve the circuit problem by breaking it to parallel and series units. Apply Ohm's law accordingly. If the units are neither in series nor parallel, then use Kirchoff's law.

Problem 4: [http://math.fullerton.edu/mathews/n2003/kirchoffmod.html]
Find the system of linear equations for the following circuit:



SOLUTIONS

Problem 1:

(a) Draw a Gaussian cylindrical surface of radius s and length l.
Inside the cylinder:

$$Q_{enc} = \int \rho dV = \int (kr)(rdrd\phi dz) = \int_0^s kr^2 dr \int_0^{2\pi} d\phi \int_0^l dz = \frac{2}{3}\pi kls^3$$

$$\int \boldsymbol{E}.d\boldsymbol{A} = E(2\pi sl)$$

$$\rightarrow \boldsymbol{E} = \frac{k}{3\epsilon_0} s^2 \hat{\boldsymbol{s}}$$

Outside the cylinder:

$$Q_{enc} = \int \rho dV = \int (kr)(rdrd\phi dz) = \int_0^R kr^2 dr \int_0^{2\pi} d\phi \int_0^l dz = \frac{2}{3}\pi klR^3$$

$$\int \boldsymbol{E}.d\boldsymbol{A} = E(2\pi sl)$$

$$\rightarrow \boldsymbol{E} = \frac{k}{3\epsilon_0}\left(\frac{R^3}{s}\right)\hat{\boldsymbol{s}}$$

(b) Use principle of superposition: a charged object with a hole produces the same field as the combination of the charged object without hole and an oppositely charged object the size of the hole, i.e., $E_{obj.with\ hole} = E_{obj.without\ hole} + E_{hole}$

$$E_{without\ hole} = \frac{k}{3\epsilon_0}\frac{R^3}{d}$$

$$Q_{hole} = \int (-\rho)dV = \int (-kr)(r^2 dr sin\theta d\theta d\phi) = -4\pi k \int_0^r r^3 dr = -\pi k r^4$$

$$E_{hole} = \frac{Q_{hole}}{\epsilon_0 A} = \frac{-\pi k r^4}{\epsilon_0 4\pi d^2}$$

$$\rightarrow \boldsymbol{E}_{with\ hole} = \frac{k}{\epsilon_0}\left(\frac{R^3}{3d} - \frac{r^4}{4d^2}\right)\hat{\boldsymbol{s}}$$

(c) Approach 1: use force:

$$Work = \int Fdx = \int QEdx = \int_d^R \frac{Qk}{\epsilon_0}\left(\frac{R^3}{3x} - \frac{r^4}{4x^2}\right)dx$$

Approach 2: use potential and potential energy:

$$Work = -\Delta U = -Q\Delta V = -Q\left(-\int Edx\right) = \int QEdx$$

Problem 2:

(a) $\sigma_a = -\frac{q_a}{4\pi a^2}, \sigma_b = -\frac{q_b}{4\pi b^2}, \sigma_R = \frac{q_a+q_b}{4\pi R^2}$

(b) $\boldsymbol{E}_{out} = \frac{1}{\epsilon_0}\frac{q_a+q_b}{4\pi r^2}\hat{\boldsymbol{r}}$

(c) $\boldsymbol{E}_a = -\frac{q_a}{4\pi\epsilon_0 r_a^2}\hat{\boldsymbol{r}}_{\boldsymbol{a}}$, $\boldsymbol{E}_b = -\frac{q_b}{4\pi\epsilon_0 r_b^2}\hat{\boldsymbol{r}}_{\boldsymbol{b}}$ where r_a, r_b are the distance away from the center of each cavity.

(d) The field in the conducting material is zero, so the charges do not “know” of each other’s presence. Therefore, $F = 0$.

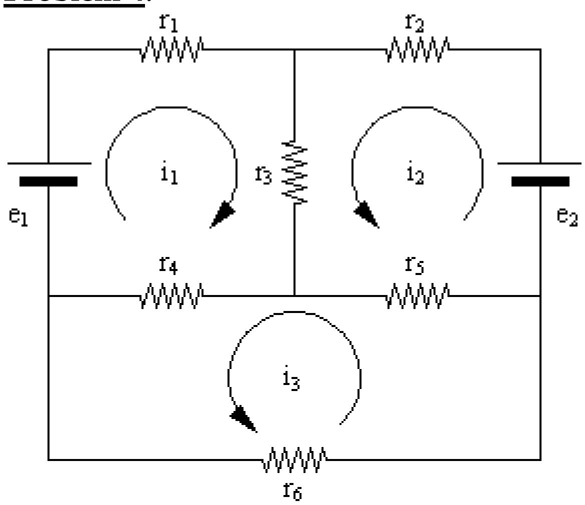
(e) σ_R and E_{out} would change.

Problem 3:
Treat the system as two capacitors in series.
(a) $E = \frac{\sigma}{\epsilon} = \frac{\sigma}{\kappa\epsilon_0}$, substitute in the corresponding value of dielectric constant for each slab.
(b) $V = V_1 + V_2 = E_1 a_1 + E_2 a_2 = \frac{\sigma}{\epsilon_0}\left(\frac{a_1}{\kappa_1} + \frac{a_2}{\kappa_2}\right)$
(c) Charge stored in capacitor: $Q = VC$, so when the capacitor is half discharged, its voltage also drops by half.

$$V = V_0 e^{-t/RC} = \frac{1}{2}V_0 \rightarrow \quad e^{-t/RC} = \frac{1}{2} \rightarrow \quad t = RC\ln 2 = R(C_1 + C_2)\ln 2$$

where $C_i = \epsilon_i A/a_i$ $(i = 1, 2)$.

Problem 4:



Loop 1: $e_1 - i_1(r_1 + r_3 + r_4) - i_2 r_3 - i_3 r_4 = 0$
Loop 2: $e_2 - i_2(r_2 + r_3 + r_5) - i_1 r_3 + i_3 r_5 = 0$
Loop 3: $i_3(r_4 + r_5 + r_6) + i_1 r_4 - i_2 r_5 = 0$