NAME: ______________________________________ SECTION: ____________

PHYSICS 7B – QUIZ 3

1. A square loop of wire (side a) lies on a table, a distance s from a very long straight wire, which carries a current I.
   (a) Find the magnetic flux through the loop
   (b) If we now pull the loop directly *away* from the wire at speed v, what emf is generated? What is the direction of the current in the loop?
   (c) What if the loop is pulled to the *right* at speed v, instead of away?

2. A uniform, time-dependent magnetic field B(t), pointing straight up, fills a circular region. Find the induced electric field.

3. A capacitor C is charged up to a potential V and connected to an inductor L. At t=0 the switch is closed. Write the differential equation to find the current in the circuit as a function of time. How does your answer change if a resistor R is included in series with C and L? (Hint: use Kirchhoff's law)

Quiz 3:

#1: (a) $B(r) = \frac{\mu_0 I}{2\pi r}\hat{\varphi}$ (r: distance away from the wire).

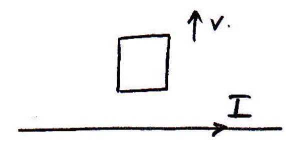
$$\Rightarrow \Phi = \int B \cdot da = \int_s^{a+s} B\cdot(a\,dr) = \frac{\mu_0 I}{2\pi} a \int_s^{a+s} \frac{dr}{r} = \boxed{\frac{\mu_0 I a}{2\pi}\ln\left(\frac{s+a}{s}\right)}.$$

(b) $$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{\mu_0 I a}{2\pi}\frac{d}{dt}\left(\ln\left(\frac{s+a}{s}\right)\right)$$

$\frac{ds}{dt} = v$ (pulling away from wire).

$$\Rightarrow \frac{d}{dt}\left(\ln\left(\frac{s+a}{s}\right)\right) = \frac{d}{dt}\left(\ln(s+a)\right) - \frac{d}{dt}(\ln s) = \frac{1}{s+a}\frac{ds}{dt} - \frac{1}{s}\frac{ds}{dt}$$

$$= \frac{v}{s+a} - \frac{v}{s} = -\frac{va}{s(s+a)}.$$

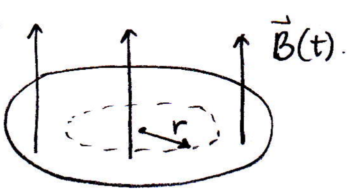


$$\Rightarrow \boxed{\mathcal{E} = \frac{\mu_0 I a^2 v}{2\pi s(s+a)}}$$

Induced current is counterclockwise

(c) $\frac{d\Phi}{dt} = 0$ (B & A are constant) $\Rightarrow \boxed{\mathcal{E} = 0}$

#2:

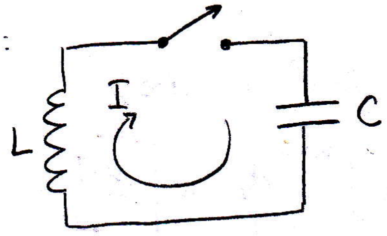


$\vec{E}$ points in the circumferential direction (like $\vec{B}$ inside a long straight wire carrying uniform $\vec{J}$) Draw an Amperian loop radius r, apply Faraday's law

$$V = \oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi}{dt} \Rightarrow E(2\pi r) = -\frac{d}{dt}(\pi r^2 B(t)) = -\pi r^2 \frac{dB}{dt}.$$

$$\Rightarrow \boxed{\vec{E} = -\frac{r}{2}\frac{dB}{dt}\hat{\phi}}.$$

#3:



$$\frac{Q}{C} + L\frac{dI}{dt} = 0.$$

$$I = \frac{dQ}{dt} \Rightarrow \boxed{\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0}$$

To solve this eqn for I: $\frac{d^2Q}{dt^2} = -\frac{1}{CL}Q = -\omega^2 Q \quad (\omega = \frac{1}{\sqrt{CL}})$.

$\rightarrow Q = A\cos\omega t + B\sin\omega t$ (general solution for this form of differential eq.).

At $t=0$: $\sin\omega t = 0$, $Q = CV = A\cos(0) \Rightarrow A = CV$.

$$I = \frac{dQ}{dt} = -\omega A\sin\omega t + \omega B\cos\omega t$$

At $t=0$: $I = 0$ (opened switch) $\Rightarrow B = 0$.

$$\Rightarrow \boxed{I(t) = -\omega CV\sin\omega t}$$

* With resistor: $\frac{Q}{C} + IR + L\frac{dI}{dt} = 0 \Rightarrow \boxed{\frac{Q}{C} + R\frac{dQ}{dt} + L\frac{d^2Q}{dt^2} = 0}$