Answers to selected problems

Worksheet E1

1. (a) $k_s(D-L)-\frac{Q^2}{4\pi\epsilon_0 D^2}=0 \rightarrow k_s(D-L)D^2-\frac{Q^2}{4\pi\epsilon_0}=0$
   (b) same
   (c) $k_s(D-L)D^2+\frac{Q^2}{4\pi\epsilon_0}=0$

4. $F=\frac{1}{4\pi\epsilon_0}\ q\ \int_d^{d+L}\frac{\lambda dx}{x^2}=\frac{1}{4\pi\epsilon_0}\frac{qQ}{L}\left(\frac{1}{d}-\frac{1}{L+d}\right)=\frac{1}{4\pi\epsilon_0}\frac{qQ}{d(d+L)}$

5.

$$Q=\int_0^L \lambda(x)dx=\int_0^L \frac{\lambda_0}{L}xdx=\frac{\lambda_0}{L}\frac{L^2}{2}=\frac{\lambda_0 L}{2}$$

In this problem, the charge distribution depends on x, so if we integrate from d to d+L like in (4), then $Q=\frac{\lambda_0}{2}(L+2d)$, which is not the correct value. Thus, the integration limits must be 0 and L.

$$F=\frac{q}{4\pi\epsilon_0}\int_0^L \frac{\lambda_0 xdx}{L(x+d)^2}$$

6. (a) $Q=\pi R\lambda$
   (b) The y-component of the force is cancelled because of symmetry, so only the x-component survives ($F_x=F\cos\theta$).

$$F=-\frac{q}{4\pi\epsilon_0}\,2\left(\int_0^{\frac{\pi}{4}}\frac{R\lambda d\theta}{R^2}\cos\theta\right)=-\frac{q}{2\pi\epsilon_0}\frac{\lambda}{R}$$

Worksheet E2

Electric field in electricity is just like $g\ (=9.8m/s^2)$ in gravity, a quantity that measures the strength to attract (or repel) of some charged object.
Pictorially, they are tracks that a second charge would follow to travel toward or away from the charged object which emanates the field.
The stronger the magnitude of the field, the denser the lines you draw to represent the field.
The direction of the field lines depend on both the sign of the charge and the symmetry of the charged object.

1. (a) Because the charges have opposite signs, the field is not cancelled in the middle but doubles in magnitude and points downward.

3. The x-component of the electric force is 0 because of symmetry, only the y-component contributes to balancing the point charge. If the line has finite length 2L (the point 0 is directly above the point charge), then:

$$mg = \int dF_y = \int dF \cos\theta = \int dF \frac{d}{\sqrt{d^2 + x^2}} = \ kq\int \frac{d}{\sqrt{d^2 + x^2}} \frac{dQ}{(d^2 + x^2)}$$

$$= kq\lambda d \int_{-L}^{L} \frac{dx}{(d^2 + x^2)^{\frac{3}{2}}} = kq\lambda d \frac{1}{a^2} \frac{2L}{\sqrt{L^2 + d^2}}$$

But in this problem we can assume that the line is infinitely long, and use the second equation in the box on page 52:

$$E = \frac{\lambda}{2\pi\epsilon_0} \frac{\hat{r}}{r}$$

So:

$$mg = \frac{q\lambda}{2\pi\epsilon_0} \frac{1}{d} \rightarrow d = \frac{q\lambda}{2\pi\epsilon_0} \frac{1}{mg}$$

4. Use force components just like in Phys 7A:

$$T \sin\theta = F_Q; \qquad T \cos\theta = mg$$

$$\rightarrow \tan\theta = \frac{F_Q}{mg} = \frac{qE}{mg}$$

For each part use the corresponding E given in the box to the right, with $r = D$.

5. Again, because the rods have opposite charges, the electric field at O is not cancelled but doubled in strength and points to the right. First find the electric field due to the positive rod, then multiply by 2 to get the total field (since they both have density λ and length L).

$$E_+ = \int_0^L \frac{1}{4\pi\epsilon_0} \frac{\lambda dx}{(x + d)^2} = \frac{1}{4\pi\epsilon_0} \int_d^{L+d} \lambda \frac{dx}{x^2} = \frac{1}{4\pi\epsilon_0} \frac{\lambda L}{d(d + L)}$$