Physics 8B - **Special Relativity**

Two equations you need to know for the second midterm (April 15, 2010):

1. **Time dilation**: time goes slower in the moving frame Therefore, the time period in the moving frame is shorter than the time period measured in static frame (Earth's frame)

$$\Delta t' = \Delta t \sqrt{1 - \frac{v^2}{c^2}}$$

2. **Length contraction**:
- If the object has a length (e.g. a rod) and it is moving along its long side, then the object's length is shortened in its frame. The other dimensions of the objects are unchanged.
- Likewise, the distance the object travels is also shorter in its frame than in the static (Earth's) frame.

$$\Delta x' = \Delta x \sqrt{1 - \frac{v^2}{c^2}}$$

Define:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Notice that since $\frac{v}{c} < 1$, the denominator is always less than 1, thus γ is always greater than 1. When you're confused whether you should multiply or divide a quantity by γ, just think of how it takes less time to travel and the distance seems shorter when you increase your speed. (Of course this is not special relativistic effect, just normal Newtonian mechanics and psychological perception, but it can help with remembering the right formula to use. Special relativity is not observed in daily life because common velocities are much smaller than c, so $\gamma \approx 1$.)

Two important conceptual points that the professor might ask you in the exam:
- Speed of light, c, is constant in **all** frame. Light **never** travels faster than $c = 3 \times 10^8 m/s$.
- Space and time change from frame to frame.

Example:

1. Problem 32 (Chap. 33, p.606):
Solution:
(a) First, find the velocity of the ship. The long way is to convert the distance of 4 light years into meter, then divide it by the given time (also converted to m/s). The short way is to use the light year unit. Observe that light takes 4 years to travel that distance, and the space ship takes 5 years to travel the same distance, so

$$\frac{v}{c} = \frac{4}{5}$$

Then:

$$\gamma = \left(1 - \left(\frac{4}{5}\right)^2\right)^{-\frac{1}{2}} = \frac{1}{\sqrt{1 - \frac{16}{25}}} = \frac{1}{\sqrt{\frac{9}{25}}} = \frac{5}{3}$$

Time should go slower on the spaceship, so the journey takes less time to complete than the time measured on Earth. Therefore you want to divide the Earth's time by γ.

$$\Delta t' = \frac{\Delta t}{\gamma} = 5\left(\frac{3}{5}\right) = 3 \text{ (years)}$$

(b) The distance the space ship has travelled, as measured in its frame:

$$\Delta x' = v\Delta t' = \frac{4}{5}c\ (3\ years) = \frac{12}{5}\ light\ years$$

Another way of doing it: since the space ship takes less time to travel, in its frame, it would feel that the distance it travels is less than the actual distance (length contraction), so we should divide the Earth-measured distance by γ:

$$\Delta x' = \frac{\Delta x}{\gamma} = 4\ lightyears * \left(\frac{3}{5}\right) = \frac{12}{5}\ light\ years$$

Two different methods lead to the same answer: a good way to check!

2. **<u>Problem 34 (Chap. 33, p. 607)</u>**

Solution:

The tube is moving, so time in its frame is shorter than time in Earth's frame, we divide the given time by γ:

$$\Delta t' = \frac{\Delta t}{\gamma} = \Delta t\sqrt{1 - \frac{v^2}{c^2}} = 6.67\ minutes * \sqrt{1 - 0.8^2}$$

$$= \frac{20}{3} minutes * \frac{3}{5} = 4\ minutes$$

This is 2 half lives. So the number of atoms left is:

$$\frac{1000}{2^2} = 250\ (atoms)$$