Summary of Chapters 6 and 7 in Pedrottis

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Energy quantization:

- Of EM fields: $E_n = h\nu\left(n + \frac{1}{2}\right)$: the ground state (vacuum, n = 0) has a finite energy
- In matter: energy of a bound electron in hydrogen atom is quantized: $E_n = -\frac{13.6eV}{n^2}$ (free electron has no quantized energy)

Thermal equilibrium:

- Atoms in thermal equilibrium (TEq):
  Giving an assembly of atoms in TEq at a temperature T, the likelihood P_i that a given atom is in one of the states of energy E_i is given by the Boltzmann distribution:
  $$P_i = P_1 e^{-\frac{E_i - E_1}{k_B T}}$$
  where i = 1 indicates the ground state.
- EM waves in TEq:
  "Spectral exitance" of a blackbody is the power per unit area per unit wavelength interval emitted by the blackbody, and it increases with the absolute temperature at each wavelength, i.e., $M_\lambda \propto \frac{P}{A\lambda} \propto f(T_\lambda)$
  $$M_\lambda = \frac{2\pi hc^2}{\lambda^5}\left(\frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}\right)$$
  Taking $\frac{dM_\lambda}{d\lambda} = 0$ to find the relationship between T and the wavelength at max exitance gives Wien displacement law:
  $$\lambda_{max} T = \frac{hc}{5k_B} = 2.898\, mm\, K$$
  Total exitance over all wavelength: Stefan-Boltzmann law:
  $$M = \int_0^\infty M_\lambda d\lambda = \sigma T^4$$

Einstein's theory of light-matter interaction:

For the absorption or emission of photon with energy $E_2 - E_1 = h\nu$, the rate of occurrence per unit volume is:

- Stimulated absorption (by incident light): $R = B_{12} g(\nu)\left(\frac{I}{c}\right) N_1$
- Stimulated emission (by incident light): $R = B_{21} g(\nu)\left(\frac{I}{c}\right) N_2$
  where I is the irradiance of incident light, $g(\nu)$ is the lineshape function, N is the population density of the level involved in the transition.
- Spontaneous emission (no incident light): result of interaction with the EM vacuum: $R = A_{21} N_2$

Relationship between Einstein A & B coefficients:

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}; \quad B_{12} = B_{21}$$

Characteristics of laser light:

- Monochromatic
- Coherent
- High degree of beam directionality
- High irradiance (power per unit area), i.e., intense
- High focusability
- Small pulse: the laser output can be turned on and off in very short time periods (a few fs)

Light interference:

- Two beams: $I = I_1 + I_2 + I_{12} = I_1 + I_2 + 2\sqrt{I_1 I_2}\langle\cos\delta\rangle$ (time average)
  where $\delta = k(s_2 - s_1) + \phi_2 - \phi_1$ is the phase difference between the two E fields, and it is time-independent for monochromatic light, i.e., $\langle\cos\delta\rangle = \cos\delta$
- Constructive interference: $\delta = 2m\pi \rightarrow \cos\delta = 1$
  Destructive interference: $\delta = (2m+1)\pi \rightarrow \cos\delta = -1$
- Visibility = $(I_{max} - I_{min})/(I_{max} + I_{min})$
- Young's double slit experiment
- Thin film interference: in case of normal incidence, the reflection coefficient is:
  $$r = \frac{1-n}{1+n} \qquad \left(n = \frac{n_2}{n_1}\right)$$
- Fringes of equal thickness (Fizeau fringes): fringes produced by a variable-thickness film (!)
  $$2n_f t + \Delta_r = \begin{cases} m\lambda & \text{bright} \\ \left(m + \frac{1}{2}\lambda\right) & \text{dark} \end{cases}$$
  where $\Delta_r = \lambda/2$ or 0, depending on whether there is an induced phase shift of the reflected rays.
- Newton's rings: for a lens of radius of curvature R on a flat optical surface, the relationship between the radius of the m^{th} dark ring and its corresponding air-film thickness is:
  $$R = \frac{r_m^2 + t_m^2}{2t_m}$$

Stokes relations:

Reflection and transmission coefficients for E fields incident on the interface between 2 media:

$$tt' = 1 - r^2; \quad r = e^{i\pi}r' = -r'$$

Amplitudes of reflected beams for rays incident from either side are the same in magnitude but differ by a π phase shift.