14.2. (a) $\vec{E} = E_0 \cos(kz-\omega t)\hat{x} - E_0 \cos(kz-\omega t)\hat{y}$.

$\Rightarrow \tilde{E}_0 = E_0\begin{bmatrix}1\\-1\end{bmatrix} \rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$: LP at $-45°$.

(b) $\vec{E} = E_0 \sin 2\pi\left(\frac{z}{\lambda}-\nu t\right)\hat{x} + E_0 \sin 2\pi\left(\frac{z}{\lambda}-\nu t\right)\hat{y}$

$\begin{bmatrix}1\\1\end{bmatrix} \rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$: LP at $45°$.

(c) $\vec{E} = E_0 \sin(kz-\omega t)\hat{x} + E_0 \sin\left(kz-\omega t-\frac{\pi}{4}\right)\hat{y}$

$\Rightarrow \tilde{E} = \left(E_0\hat{x} + E_0 e^{-i\pi/4}\hat{y}\right)e^{i(kz-\omega t)}$

$\Rightarrow \tilde{E}_0 = E_0\begin{bmatrix}1\\e^{-i\pi/4}\end{bmatrix} \Rightarrow A\begin{bmatrix}1\\\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\end{bmatrix}$ (14.9).

$= A\begin{bmatrix}1\\\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\end{bmatrix} = A\begin{bmatrix}1\\\frac{1}{\sqrt{2}}(1-i)\end{bmatrix}$

$\rightarrow$ normalize to get A: $A\left(1+\frac{1}{2}(1-i)(1+i)\right) = 1 \rightarrow A = \frac{1}{\sqrt{2}}$.

(14.10): $\tan 2\alpha = \dfrac{2E_{0x}E_{0y}\cos\varepsilon}{E_{0x}^2 - E_{0y}^2}$

$E_{0x} = E_{0y} = 1 \Rightarrow \tan 2\alpha \rightarrow \infty \Rightarrow 2\alpha = \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{4}$.

$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\\frac{1}{\sqrt{2}}(1-i)\end{bmatrix}$; REP at $45°$.

(d) $\vec{E} = E_0\cos(kz-\omega t)\hat{x} + E_0\cos\left(kz-\omega t+\frac{\pi}{2}\right)\hat{y}$

$\Rightarrow \tilde{E} = \left(E_0\hat{x} + E_0 e^{i\pi/2}\hat{y}\right)e^{i(kz-\omega t)}$

$\Rightarrow \tilde{E}_0 = E_0\begin{bmatrix}1\\e^{i\pi/2}\end{bmatrix} \Rightarrow A\begin{bmatrix}1\\\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\end{bmatrix} = A\begin{bmatrix}1\\i\end{bmatrix} \rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}$

$\rightarrow$ LCP

14.16: Determine the state of pol. of CP light after it is passed normally through

(a) a QWP

(b) an eighth-wave plate

Use the matrix method.

(a) LCP: $\begin{pmatrix}1\\i\end{pmatrix}$, RCP: $\begin{pmatrix}1\\-i\end{pmatrix}$

QWP, SA vertical $\begin{pmatrix}1&0\\0&i\end{pmatrix}$, SA horizontal $\begin{pmatrix}1&0\\0&-i\end{pmatrix}$. with pre-factors.

$$\underbrace{\begin{pmatrix}1&0\\0&i\end{pmatrix}\begin{pmatrix}1\\i\end{pmatrix}=\begin{pmatrix}1\\-1\end{pmatrix};\quad \begin{pmatrix}1&0\\0&-i\end{pmatrix}\begin{pmatrix}1\\-i\end{pmatrix}=\begin{pmatrix}1\\-1\end{pmatrix}}_{\text{LP at } -45^\circ}\qquad \underbrace{\begin{pmatrix}1&0\\0&i\end{pmatrix}\begin{pmatrix}1\\-i\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix};\quad \begin{pmatrix}1&0\\0&-i\end{pmatrix}\begin{pmatrix}1\\i\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}}_{\text{LP at } 45^\circ}$$

(b) An eighth-wave plate is a phase retarder that introduces a relative phase of $|\varepsilon_y-\varepsilon_x| = \pi/4$.

The Jones matrix is:

$$M_{1/8} = \begin{pmatrix}1&0\\0&e^{\pm i\pi/4}\end{pmatrix} \qquad (\pm:\ \varepsilon_y>\varepsilon_x \text{ or } \varepsilon_y<\varepsilon_x).$$

$$\begin{pmatrix}1&0\\0&e^{\pm i\pi/4}\end{pmatrix}\begin{pmatrix}1\\i\end{pmatrix} = \begin{pmatrix}1\\e^{3i\pi/4}\end{pmatrix} \text{ or } \begin{pmatrix}1\\e^{i\pi/4}\end{pmatrix} \qquad (i=e^{i\pi/2}).$$

$$\begin{pmatrix}1&0\\0&e^{\pm i\pi/4}\end{pmatrix}\begin{pmatrix}1\\-i\end{pmatrix} = \begin{pmatrix}1\\e^{-3i\pi/4}\end{pmatrix} \text{ or } \begin{pmatrix}1\\e^{-i\pi/4}\end{pmatrix}$$

Express in terms of $\begin{pmatrix}A\\B+iC\end{pmatrix} = \begin{pmatrix}1\\\frac{1}{\sqrt{2}}(\pm1\pm i)\end{pmatrix}$: EP.

14.22. A QWP is placed between crossed polarizers s.t. the angle b.w. the polarizer TA of the 1st polarizer & the QWP fast axis is θ.
How does the pol. axis of the emergent light vary as a function of θ?

Take the fast axis of the QWP along x-axis.

The Jones matrix for QWP (FA-x) : $\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} = M_{QWP}$

Light emerge from 1st polarizer: $\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$.

Crossed polarizers $\Rightarrow$ TA of 2nd polarizer is at $\alpha = 90° + \theta$. $\Rightarrow$ $\begin{cases} \cos\alpha = -\sin\theta \\ \sin\alpha = \cos\theta \end{cases}$

Use Eq. (14-15) to get Jones matrix of 2nd polarizer:

$$M_2 = \begin{pmatrix} \cos^2\alpha & \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha & \sin^2\alpha \end{pmatrix} = \begin{pmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{pmatrix}$$

$\rightarrow$ Jones vector of output light:

$$M_2 M_{QWP} \underbrace{\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}}_{Amp=1} = \begin{pmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

$$= \begin{pmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{pmatrix} \begin{pmatrix} \cos\theta \\ i\sin\theta \end{pmatrix}$$

$$= \begin{pmatrix} \sin^2\theta\cos\theta - i\sin^2\theta\cos\theta \\ -\sin\theta\cos^2\theta + i\sin\theta\cos^2\theta \end{pmatrix}$$

$$= (1-i)\sin\theta\cos\theta \underbrace{\begin{pmatrix} \sin\theta \\ -\cos\theta \end{pmatrix}}_{Amp=1.}$$

$$\Rightarrow I = I_0 \sin^2\theta\cos^2\theta \, |1-i|^2 = 2\sin^2\theta\cos^2\theta \, I_0 .$$