

Physics 110 B Final Exam Spring 2012

Partial credit will be given, so show your reasoning carefully. The number of points for each problem is listed at the left. You are permitted three sheet of notes, written on both sides. Numerical answers will be judged correct if within 1% of the right answer.

1. [5] A neutron has a mean lifetime of 882 s and a mass of 1.67×10^{-27} kg. What would be the total energy of a neutron that could travel a light year before it decayed? You may assume that the speed of the neutron is very close to that of light. A year has 3.16×10^7 s.
2. [15] (Griffiths Prob 12.16—the twin paradox). Alice takes off from Earth in a rocket on her 21st birthday at a speed of $\frac{4}{5}c$. When she reaches her destination, she hops on a rocket moving at the same speed in the opposite direction, arriving home on her 39th birthday.
 - a) [3] How old is her twin brother Bob when she returns home?
 - b) [3] How far away was her destination as measured in Earth's coordinate system?
 - c) [9] How old did Alice think Bob was just before she jumped onto the return rocket? How old just after?
3. [10] A nonrelativistic photon can scatter off a free electron in a process called Thomson scattering. The cross section for this process, σ , is given by the ratio of the scattered power to the incident flux, $\sigma = P/S$. By considering an isolated electron in a plane electromagnetic wave, determine the Thomson scattering cross section in terms of fundamental constants: c , ϵ_0 , the electron charge, e , and the electron mass, m_e . You do not need to evaluate this numerically; it is about 10^{-28} m². Assume that the motion of the electron is non-relativistic, so the magnetic force may be neglected.
4. [10] A cylindrical laser with a radius w emits light into a beam with a half-angle θ . The power of the laser is P .
 - a) [3] What is the intensity of the emitted light? (This is called the radiance in the text.)
 - b) [3] An observer at a distance D observes the laser emission with a telescope that is able to resolve the laser, so she is able to determine both the flux from the laser, S_{obs} , and the radius, w . What intensity does she measure?
 - c) [4] Show that the intensities in parts (a) and (b) are the same.
5. [10] A thin soap film, with an index of refraction of 1.4, has air on both sides.
 - a) [3] Why would the soap film appear black if its thickness approached zero?
 - b) [7] Light reflected from the film at normal incidence has a spectrum that has three peaks in the visible part of the spectrum, at 666 nm, 545 nm, and 462 nm. How thick is the film? For this problem, 5% accuracy is satisfactory.

6. [15] One of the methods for making 3D movies depends on circularly polarized light. When you wear the proper glasses, one eye sees only right circularly polarized (RCP) light and the other sees only left circularly polarized (LCP) light. The projector alternates rapidly between the right and left circularly polarized images, creating the 3D effect.
- a) [12] Design a pair of 3D glasses using quarter wave plates and linear polarizers. For simplicity, assume that the QWPs have the same alignment for each eye; state the alignment you select. You must specify the relative location of the QWP and polarizer, and the orientation of the polarizer, for each lens such that the left eye sees only LCP light and the right eye only RCP light. Design your scheme with the aid of Jones matrices. (The three Jones matrices we discussed in class are given below; you need at most two of them.) Label your coordinate system clearly.
 - b) [3] For RCP light entering the right eye, what are the three Stokes parameters (S_1 , S_2 , S_3) just before the light reaches the lens and just after? Express your answers in terms of S_0 .
7. [25] Diffraction due to two infinitely long slits of width b and separation a .
- a) [15] Derive the Fraunhofer diffraction pattern due to these two slits. Your answer should be an expression for I/I_0 , where I_0 is the intensity at the center of the diffraction pattern.
 - b) [3] For light of wavelength λ , what is the condition on the distance, R_0 , between the aperture plane with the slits and the screen on which the diffraction is observed in order for the diffraction to be of the Fraunhofer type?
 - c) [7] (Pedrotti 11.15) Using green light of wavelength 546.1 nm and slits of width 0.100 mm, it is observed that the fourth order interference maxima are missing from the pattern. What is the slit separation, a ?
8. [10] Design a Fresnel zone plate that magnifies the intensity of an incident plane wave of wavelength 500 nm by a factor 36 at a distance of 1 m. Give numerical values for the relevant radii.

Jones matrices: $M = \begin{bmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 \\ 0 & e^{i(\delta_y - \delta_x)} \end{bmatrix}, \quad \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix}$