Physics 110B Final Solution – Spring 2012

1. [5]

$$E = mc^2 = \gamma m_0 c^2 = \frac{\Delta t}{\Delta t'}\, m_0 c^2$$
$$= \frac{3.16 \times 10^7 s}{882\ s}\ (1.67 \times 10^{-27}\ kg)\left(3 \times 10^8 \frac{m}{s}\right)^2 = \mathbf{5.38 \times 10^{-6} J}$$

2.
(a) [3]

$$\beta = \frac{v}{c} = \frac{4}{5}$$
$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{5}{3}$$

The time it takes Alice to travel from Earth to the destination in Alice's frame: (39 – 21)/2 = 9 (years). Thus, during Alice's trip, Bob has aged by:

$$\Delta t = \gamma \Delta t' = \frac{5}{3}(9\ years)(2) = \ 30\ years$$

So Bob is **51 years old** when Alice returns.
(b) [3]

$$\Delta x = \frac{v\Delta t}{2} = \frac{4}{5}c(15\ years) = \mathbf{12\ lightyears}$$

(c) [9]
Just before she jumped onto the return rocket:

$$\Delta t'' = \frac{\Delta t'}{\gamma} = \frac{9\ years}{\frac{5}{3}} = 5.4\ years$$

So Alice thought that Bob was **26.4 years old**.
Just after she jumped onto the return rocket:
Since the return frame has a negative velocity (opposite that of the departing frame), the time coordinate of the jump is given by:

$$\tilde{t} = \gamma\left(\mathrm{t} + \frac{\mathrm{v}}{\mathrm{c}^2}x\right) = \frac{5}{3}\left(15\ years + \frac{4}{5}\frac{12\ ly}{c}\right) = \ 41\ years$$

Thus she thought Bob has aged by:

$$\Delta\tilde{t}' = \frac{\Delta\tilde{t}}{\gamma} = 24.6\ years$$

i.e., he was **45.6 years old**.

3. [10]
For a plane EM wave, the instantaneous flux is given by:

$$S = \ \epsilon_0 c E^2$$

Use Larmor's formula to get the power radiated by the electron:

$$P = \frac{\mu_0 e^2 a^2}{6\pi c} = \frac{e^2}{6\pi}\left(\frac{1}{\epsilon_0 c^3}\right)(a^2)$$

$$a = \frac{F}{m_e} = \frac{eE}{m_e}$$
$$\rightarrow P = \frac{e^4 E^2}{6\pi\epsilon_0 c^3 m_e^2}$$
Thus, the scattering cross section is:
$$\sigma = \frac{P}{S} = \boldsymbol{\frac{e^4}{6\pi\epsilon_0^2 c^4 m_e^2}}$$

4.
(a) [3]
$$I_0 = \frac{S_0}{\Omega} = \frac{P}{\Omega A_0} = \frac{P}{\pi^2\theta^2 w^2}$$
(b) [3]
Angular size of the laser beam: $\theta_I = w/D$. Thus:
$$I = \frac{S_{obs}}{\Omega} = \frac{S_{obs}}{\pi\theta_I^2} = \frac{S_{obs}D^2}{\pi w^2}$$
(c) [4]
Because the power is emitted into angle θ, the flux that reaches D is:
$$S_{obs} = \frac{P}{\pi(\theta D)^2}$$
$$\rightarrow I = \frac{S_{obs}D^2}{\pi w^2} = \frac{P}{\pi\theta^2 D^2}\frac{D^2}{\pi w^2} = \frac{P}{\pi^2\theta^2 w^2} \qquad \blacksquare$$

5.
(a) [3]
For a soap film, due to the phase shift of the reflection at the air-soap interface, constructive interference occurs when the path difference of 2 reflected rays is a half integral number of wavelengths, and destructive interference occurs when the path difference is an integral number of wavelengths. Thus, if the thickness approaches 0, i.e., the path difference approaches 0, then there is only destructive interference and the film appears black.
(b) [7]
$$2nt = \left(m_1 + \frac{1}{2}\right)\lambda_1 = \left(m_2 + \frac{1}{2}\right)\lambda_2 = \left(m_3 + \frac{1}{2}\right)\lambda_3$$
$$\frac{m_1 + 0.5}{m_2 + 0.5} = \frac{545}{666} = 0.82; \qquad \frac{m_1 + 0.5}{m_3 + 0.5} = \frac{462}{666} = 0.69$$
After some trials and errors, we get $(m_1, m_2, m_3) = (4, 5, 6)$. Therefore, the thickness of the film is:
$$t = \frac{4.5\lambda_1}{2n} = \frac{4.5(666\ nm)}{2(1.4)} = \mathbf{1.07\ \mu m}$$

6.
(a) [12]
What we have:
RCP light: $\begin{pmatrix} 1 \\ -i \end{pmatrix}$, LCP light: $\begin{pmatrix} 1 \\ i \end{pmatrix}$

Let the light first go through a QWP with vertical SA, i.e., $e^{i(\delta_y-\delta_x)} = i$, then through an LP whose TA is at angle θ relative to the SA of the QWP, then the total transformation is:

$$\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} = \begin{bmatrix} \cos^2\theta & i\cos\theta\sin\theta \\ \cos\theta\sin\theta & i\sin^2\theta \end{bmatrix}$$

If no LCP light passes through (the right eye lens), then:

$$\begin{bmatrix} \cos^2\theta & i\cos\theta\sin\theta \\ \cos\theta\sin\theta & i\sin^2\theta \end{bmatrix}\begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} \cos^2\theta - \cos\theta\sin\theta \\ \cos\theta\sin\theta - \sin^2\theta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\rightarrow \cos^2\theta = \cos\theta\sin\theta = \sin^2\theta = \frac{1}{2}$$

$$\rightarrow \boldsymbol{\theta = 45°}$$

Similarly, if no RCP passes through (the left eye lens), then $\boldsymbol{\theta = -45°}$

(b) [3]

For the RCP light reaching the right eye, its complete Jones vector is:

$$\tilde{\boldsymbol{E}}_0 = E_{0x}\begin{pmatrix} 1 \\ \frac{E_{0y}}{E_{0x}} e^{i\Delta\phi} \end{pmatrix} = \frac{\sqrt{I}}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$$\rightarrow E_{0x} = E_{0y} = \frac{\sqrt{I}}{\sqrt{2}} \quad \text{and} \quad \Delta\phi = -\frac{\pi}{2}$$

So its Stokes parameters just before it reaches the lens are:

$$S_1 = \langle E_{0x}^2 \rangle - \langle E_{0y}^2 \rangle = 0$$

$$S_2 = 2E_{0x}E_{0y}\cos\Delta\phi = 0$$

$$S_3 = -2E_{0x}E_{0y}\sin\Delta\phi = I = S_0$$

After transmitted through the lens, the light becomes linearly polarized:

$$\begin{bmatrix} \cos^2\theta & i\cos\theta\sin\theta \\ \cos\theta\sin\theta & i\sin^2\theta \end{bmatrix}\begin{pmatrix} 1 \\ -i \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\rightarrow \Delta\phi = \pi$$

So its Stokes parameters are: $(S_1, S_2, S_3) = (0, S_0, 0)$

7.

(a) [15]

The differential electric field at a point P on the screen is given by:

$$dE_P = \frac{E_L ds}{r} e^{i(kr-\omega t)} e^{iks\sin\theta}$$

where *ds* is a length interval of the plane wave passing through each slit, *r* is the optical-path length from *ds* to P, and $\Delta = s\sin\theta \ll r$ is the path difference (Figure 11-1, Pedrottis).

Integrate over the slit widths to obtain the total field at P:

$$E_P = \frac{E_L}{r} e^{i(kr-\omega t)} \left(\int_{-\frac{1}{2}(a+b)}^{-\frac{1}{2}(a-b)} e^{isk\sin\theta} ds + \int_{\frac{1}{2}(a-b)}^{\frac{1}{2}(a+b)} e^{isk\sin\theta} ds \right)$$

Following integration and defining

$$\beta \equiv \frac{1}{2}kb\sin\theta\,; \qquad \alpha \equiv \frac{1}{2}ka\sin\theta$$

we get:

$$E_P = \frac{E_L}{r} e^{i(kr-\omega t)} \frac{b}{2i\beta}\left(e^{i\alpha}\left(e^{i\beta} - e^{-i\beta}\right) + e^{-i\alpha}\left(e^{i\beta} - e^{-i\beta}\right)\right)$$
$$= \frac{E_L}{r} e^{i(kr-\omega t)} \frac{b}{2i\beta}(2i\sin\beta)(2\cos\alpha)$$
$$= \frac{E_L}{r} e^{i(kr-\omega t)} \frac{2b\sin\beta}{\beta}\cos\alpha$$
The amplitude of this electric field is:
$$E_0 = \frac{E_L}{r}\ 2b\left(\frac{\sin\beta}{\beta}\right)\cos\alpha$$
So the irradiance at point P in the diffraction pattern is:
$$I = \left(\frac{\epsilon_0 c}{2}\right) E_0^2 = \left(\frac{\epsilon_0 c}{2}\right)\left(\frac{2E_L b}{r}\right)^2 \left(\frac{\sin\beta}{\beta}\right)^2 \cos^2\alpha$$
$$\rightarrow \frac{I}{I_0} = \left(\frac{\sin\beta}{\beta}\right)^2 \cos^2\alpha$$
(b) [3]
Diffraction minima occur for $m\lambda = b\sin\theta \approx b\theta$, so the angular width of the central peak ($m = 1$) is $\Delta\theta = 2\lambda/b$
The width of the central maximum is:
$$W = L\Delta\theta = \frac{2L\lambda}{b}$$
$$\rightarrow L_{min} = \frac{b^2}{2\lambda} \text{ when } W = b$$
Therefore, the distance $R_0 \gg L_{min} \approx b^2/\lambda$ to have a Fraunhofer diffraction.

(c) [7]
The condition for missing orders is: $a = \left(\frac{p}{m}\right) b$
The 4th-order interference maxima are missing, i.e., $p = 4m$, hence:
$$a = 4b = 4(0.1\ mm) = \mathbf{0.4\ mm}$$

8. [10]
The intensity is increased by a factor of 36, i.e., the amplitude is 6 times the amplitude of a wholly unobstructed wavefront, so there are 3 transmitted zones, whose outer radii are:
$$R_1 = \sqrt{(1)(1)(5\times 10^{-7})} = \mathbf{0.00071\ (m)}$$
$$R_3 = \sqrt{(3)(1)(5\times 10^{-7})} = \mathbf{0.0012\ (m)}$$
$$R_5 = \sqrt{(5)(1)(5\times 10^{-7})} = \mathbf{0.0016\ (m)}$$
and inner radii are:
$$R_0 = \mathbf{0}$$
$$R_2 = \mathbf{0.001\ m}$$
$$R_4 = \sqrt{(4)(1)(5\times 10^{-7})} = \mathbf{0.0014\ (m)}$$