PHYS 110 B – Homework 10

Solutions

① $\nu_F = \frac{c}{2dn} = \frac{3\times10^8\,m/s}{2(0.2\,mm)(3.6)} = 208.33$ GHz.

$r = \frac{n_1-n_2}{n_1+n_2} = \frac{3.6-1}{3.6+1} = 0.5625$

$\alpha_{eff} = \alpha_s + \frac{1}{2d}\ln\left(\frac{1}{R_1R_2}\right) = 1\,cm^{-1} + \frac{1}{2(0.2\,mm)}\ln\left(\frac{1}{0.5625^2}\right) = 29.77\,cm^{-1}$.

$\mathcal{F} = \frac{\pi e^{-\alpha_{eff}d}}{1-e^{-2\alpha_{eff}d}} = 2.49$.

$\delta\nu = \frac{\nu_F}{\mathcal{F}} = 83.61$ GHz.

② $\nu_F = 150\,MHz$, $\delta\nu = 5\,MHz$, $n=1$

Length $d = \frac{c}{2\nu_F} = 1\,m$.

$\mathcal{F} = \frac{\nu_F}{\delta\nu} = 30$.

$\alpha_{eff} = \frac{1}{2d}\ln\left(\frac{1}{R^2}\right) = \underset{(cm^{-1})}{-\ln R} \Rightarrow \mathcal{F} = \frac{\pi e^{\ln R}}{1-e^{2\ln R}} = \frac{\pi R}{1-R^2} \Rightarrow R \simeq 0.95$

③ $\tau_p = \frac{\mathcal{F}d}{\pi c}$; $E = E_0 e^{-t/\tau_p} = \frac{1}{2}E_0 \Rightarrow t = -\tau_p \ln\left(\frac{1}{2}\right) = -\frac{\mathcal{F}d}{\pi c}\ln(1/2)$

$\Rightarrow t = -\frac{100\,(50\,cm)}{\pi c}\ln(1/2) = 36.8\,ns$.

④ $\frac{I}{I_0} = 12 = e^{\gamma d_0} \left(= e^{\gamma d} \right) \Rightarrow \gamma = \frac{1}{d_0} \ln 12 \quad (d_0 = 15 cm)$

Keep γ constant and increase d from $d_0 = 15$ cm to $d_1 = 20$ cm :

$$\frac{I}{I_0} = e^{\gamma d_1} = e^{(\ln 12)\, d_1/d_0} = 27.5$$

⑤ (a) $\nu_F = \frac{c}{2nd} = \frac{c}{2(1)(100 cm)} = 0.15$ GHz.

(b) $\delta\nu = 3.5$ GHz.

α_{loss} is $\frac{1}{2}$ the peak small-signal gain coefficient $\gamma_0(\nu_0)$

$\Rightarrow$ Number of modes $M \leqslant \frac{\delta\nu}{\nu_F} = \frac{3.5}{0.15} = 23.3 \Rightarrow M = 23.$

(c) To achieve resonance on a single mode :

$$M < 2 \rightarrow 2 > \frac{\delta\nu}{\nu_F} = \frac{\delta\nu}{c/2nd} = \frac{2nd\,\delta\nu}{c}$$

$$\Rightarrow d < \frac{c}{n\,\delta\nu} = \frac{3 \times 10^8 m/s}{(1)(3.5 \times 10^9 Hz)} = 0.086 m = 8.6 cm.$$