Homework 4

① $N = 10^{23}/cm^3 \rightarrow$ not dilute $\rightarrow$ need to account for local field effect.

$$\chi = -\frac{Ne^2}{\varepsilon_0 m}\sum_k \frac{f_k}{\omega^2 - \omega_k^2 - i\omega\gamma_k} \quad , \varepsilon = 1 + \chi .$$

$$= -\frac{Ne^2}{\varepsilon_0 m}\frac{f}{\omega^2 - \omega_0^2 + i\omega\gamma} \qquad (\hbar\omega_0 = 1eV,\ \gamma = 0.01eV,\ f = 1).$$

(a) $$\varepsilon = 1 - \frac{Ne^2}{\varepsilon_0 m}\frac{f}{\omega^2-\omega_0^2+i\omega\gamma} = 1 - \frac{Ne^2}{\varepsilon_0 m}\frac{f(\omega^2-\omega_0^2-i\omega\gamma)}{(\omega^2-\omega_0^2)^2+\omega^2\gamma^2}$$

$$\Rightarrow Re(\varepsilon) = 1 - \frac{Ne^2}{\varepsilon_0 m}\frac{f(\omega^2-\omega_0^2)}{(\omega^2-\omega_0^2)^2+\omega^2\gamma^2} = \varepsilon'$$

$$Im(\varepsilon) = \frac{Ne^2}{\varepsilon_0 m}\frac{(-\omega\gamma f)}{(\omega^2-\omega_0^2)^2+\omega^2\gamma^2} = \varepsilon''$$

(b) $n = n' + in''$, $n^2 = \varepsilon = n'^2 - n''^2 + 2in'n'' = \varepsilon' + i\varepsilon''$

$$\Rightarrow n'^2 - n''^2 = \varepsilon'$$

$$2n'n'' = \varepsilon'' \Rightarrow n'' = \frac{\varepsilon''}{2n'} \Rightarrow n'^2 - \frac{\varepsilon''^2}{4n'^2} = \varepsilon' \Rightarrow 4n'^4 - 4n'^2\varepsilon' - \varepsilon''^2$$

$$\Rightarrow n'^2 = \frac{4\varepsilon' \pm \sqrt{16\varepsilon'^2 + 16\varepsilon''^2}}{8} = \frac{\varepsilon' \pm \sqrt{\varepsilon'^2+\varepsilon''^2}}{2}.$$

$$n' = \left(\frac{\varepsilon' \pm \sqrt{\varepsilon'^2+\varepsilon''^2}}{2}\right)^{1/2}.$$

$$\Rightarrow n'' = \frac{\varepsilon''}{\sqrt{2}\left(\varepsilon' \pm \sqrt{\varepsilon'^2+\varepsilon''^2}\right)^{1/2}}$$

(2) Chartier (8.22): $E = E_0 e^{-k''z} e^{-i\omega t} e^{ik'z}$

(8.23): $I = I_0 e^{-2k''z} = I_0 e^{-\alpha z}$ ($\alpha = 2k''$: absorption coeff.)

where $k' = \frac{n'\omega}{c}$, $k'' = \frac{n''\omega}{c}$.

$\Rightarrow \alpha = \frac{2n''\omega}{c}$.

$n = n' + in'' = \sqrt{\varepsilon} = \sqrt{1+\chi' + i\chi''}$

$\Rightarrow n'^2 - n''^2 + 2in'n'' = 1 + \chi' + i\chi''$

$\Rightarrow n'^2 - n''^2 = 1 + \chi'$

$2n'n'' = \chi'' \Rightarrow n' = \frac{\chi''}{2n''}$.

$\Rightarrow \frac{\chi''^2}{4n''^2} - n''^2 = 1 + \chi'$.

$\Rightarrow \chi''^2 - 4n''^4 - 4(1+\chi')n''^2 = 0$.

$\Rightarrow n''^2 = \frac{4(1+\chi') \pm \sqrt{16(1+\chi')^2 + 16\chi''^2}}{-8} = -\frac{(1+\chi') \pm \sqrt{(1+\chi')^2 + \chi''^2}}{2}$.

$\Rightarrow \alpha = \frac{\omega\sqrt{2}}{c}\left[-1+\chi' \mp \sqrt{(1+\chi')^2 + \chi''^2}\right]^{1/2}$.

(3) (a) $I = I_0 e^{-\alpha z}$

$I = \frac{I_0}{4}$, $z = 3.42\text{cm} \Rightarrow e^{-\alpha z} = \frac{1}{4} \Rightarrow \alpha = 0.405\ m^{-1}$.

(b) $I = \frac{I_0}{100} \Rightarrow z = -\frac{1}{\alpha} \ln(0.01) = 11.37\ \text{cm}$.

(4) $\vec{P} = \varepsilon_0 \chi \vec{E}$ & $E = \frac{\sigma_b}{\varepsilon_0} = \frac{\vec{P}.\hat{n}}{\varepsilon_0} \Rightarrow \chi = 1. \Rightarrow \chi'' = Im(\chi) = 0$

$\chi = \frac{\omega_p^2}{\omega^2 + i\omega\gamma}$ ($\omega_0 = 0$ for metal) $\Rightarrow \gamma = 0$ & $\omega = \omega_p$