PHYS 110B

Homework 5

① $\tilde{\varepsilon} = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\gamma}$ for metal

(a) Griffiths (9.147): $\frac{\tilde{E}_{0R}}{\tilde{E}_{0I}} = \frac{1-\tilde{\beta}}{1+\tilde{\beta}}$, $\frac{\tilde{E}_{0T}}{\tilde{E}_{0I}} = \frac{2}{1+\tilde{\beta}}$

$\beta = \frac{\mu_1 v_1}{\mu_2 \omega} \tilde{k}_2$ & Griffiths (9.169): $\tilde{k} \approx \frac{\omega}{c}\sqrt{\tilde{\varepsilon}_r} \approx \frac{\omega}{c}\tilde{n}$

$\Rightarrow \tilde{\beta} \approx \frac{c}{\omega}\frac{\omega}{c}\tilde{n} = \tilde{n}$

$\Rightarrow \frac{\tilde{E}_{0R}}{\tilde{E}_{0I}} = \frac{1-\tilde{n}}{1+\tilde{n}}$; $\frac{\tilde{E}_{0T}}{\tilde{E}_{0I}} = \frac{2}{1+\tilde{n}}$

$\tilde{n} = \left(1 - \frac{\omega_p^2}{\omega^2 + i\omega\gamma}\right)^{1/2} = \left(1 - \frac{\omega_p^2(\omega^2 - i\omega\gamma)}{\omega^4 + \omega^2\gamma^2}\right)^{1/2}$

$\omega = \frac{2\pi c}{\lambda} = 2.98 \times 10^{15}$ Hz, $\omega_p = 2\pi \times 10^{15}$ Hz, $\gamma = 10^{13}$ Hz.

$\Rightarrow \tilde{n} \approx 0.004 + 1.856i$

$\Rightarrow \frac{\tilde{E}_{0R}}{\tilde{E}_{0I}} \approx -0.549 - 0.834i$

$\frac{\tilde{E}_{0T}}{\tilde{E}_{0I}} \approx 0.451 - 0.834i$

(b) $d \equiv \frac{1}{\mathrm{Im}(k)} = \frac{1}{\mathrm{Im}(\omega\sqrt{\tilde{\varepsilon}\mu_0\varepsilon_0})} \approx 5.435 \times 10^{-8}$ (m)

(c) $R = \left|\frac{1-\tilde{\beta}}{1+\tilde{\beta}}\right|^2 = \left|\frac{1-\tilde{n}}{1+\tilde{n}}\right|^2 = 0.997$

$\Rightarrow$ 0.003 of the original energy is absorbed.

(2) (d) $\omega = \dfrac{2\pi c}{\lambda} = 1.885 \times 10^{16}$ Hz.

$\Rightarrow \tilde{n} = \sqrt{\tilde{\varepsilon}} \approx 0.943 + 0.0000313i$

$\Rightarrow \dfrac{\tilde{E}_{0R}}{\tilde{E}_{0I}} = \dfrac{1-\tilde{n}}{1+\tilde{n}} \approx 0.0293 - 0.0000166i$

$\Rightarrow \dfrac{\tilde{E}_{0T}}{\tilde{E}_{0I}} = \dfrac{2}{1+\tilde{n}} \approx 1.029 - 0.0000166i$

(e) $d = \dfrac{1}{\mathrm{Im}(\omega\sqrt{\tilde{\varepsilon}\mu_0\varepsilon_0})} \approx 0.000508$ (m)

(f) $R = 0.00086 \Rightarrow 0.99914$ % of original energy is absorbed.

(3) $\tilde{k} = \frac{\omega}{c}\sqrt{\tilde{\varepsilon}} = \frac{\omega}{c}\sqrt{1-\frac{\omega_p^2}{\omega(\omega+i\gamma)}} = \frac{\omega}{c}\sqrt{1-\frac{\omega_p^2}{\omega^2+\gamma^2}+i\frac{\omega_p^2\gamma}{\omega(\omega^2+\gamma^2)}}$

Assume $Re(\tilde{n}) \gg Im(\tilde{n}) \Rightarrow \omega \gg \gamma$.

$\Rightarrow k = \frac{\omega}{c}\sqrt{1-\frac{\omega_p^2}{\omega^2+\gamma^2}} \Rightarrow V_{phase} = \frac{\omega}{k} = \frac{c}{\sqrt{1-\omega_p^2/\omega^2+\gamma^2}} = \frac{c}{cRe(\tilde{k})/\omega}$

$$\frac{dk}{d\omega} = \frac{d}{d\omega}\left(\frac{\omega}{c}\sqrt{1-\frac{\omega_p^2}{\omega^2+\gamma^2}}\right) = \frac{\gamma^4-\gamma^2(\omega_p^2-2\omega^2)+\omega^4}{c(\gamma^2+\omega^2)^2\sqrt{1-\omega_p^2/\omega^2+\gamma^2}}$$

$$V_{group} = \frac{d\omega}{dk} = \frac{c(\gamma^2+\omega^2)^2\sqrt{1-\omega_p^2/(\omega^2+\gamma^2)}}{\gamma^4-\gamma^2(\omega_p^2-2\omega^2)+\omega^4} = \frac{c(\gamma^2+\omega^2)^2 Re(\tilde{k})/(\omega/c)}{\gamma^4-\gamma^2(\omega_p^2-2\omega^2)+\omega^4}$$

ω	$Re(\tilde{k})(\frac{c}{\omega})$	V_p	V_g
2.98×10^{15} Hz		undef.	undef.
1.885×10^{16} Hz		undef.	undef.
$\omega=\omega_p=2\pi\times10^{15}$ Hz	0.028	35c	0.028c.

when $\omega \leq \omega_p$, the condition of $Re(\tilde{n}) \gg Im(\tilde{n})$ is not satisfied, thus V_p & V_g are ill defined.

④ For s-polarized light, the reflection coefficient is:

$$r = \frac{\tilde{n}_1 \cos\theta_i - \tilde{n}_2 \cos\theta_t}{\tilde{n}_1 \cos\theta_i + \tilde{n}_2 \cos\theta_t} \qquad (\tilde{n}_1 = 1).$$

For $\lambda = 632\text{nm}$, $\tilde{n}_2 = 0.004 + 1.856i$

$$n_2' \sin\theta_t = \sin\theta_i$$

$$\cos\theta_t = \sqrt{1-\sin^2\theta_t} = \sqrt{1-\frac{\sin^2\theta_i}{n_2'}}$$

$$\Rightarrow r = \frac{\cos\theta_i - \tilde{n}_2\left(1-\frac{\sin^2\theta_i}{n_2'}\right)^{1/2}}{\cos\theta_i + \tilde{n}_2\left(1-\frac{\sin^2\theta_i}{n_2'}\right)^{1/2}}$$

(5) 9.22 Griffiths

(a) $v = \alpha\sqrt{\lambda} \Rightarrow \frac{\omega}{k} = \alpha\sqrt{\frac{2\pi}{k}} \Rightarrow \omega = \alpha\sqrt{2\pi k}$

$(9.150): \; v_g = \frac{d\omega}{dk} = \frac{d}{dk}\left(\alpha\sqrt{2\pi k}\right) = \alpha\sqrt{2\pi}\,\frac{1}{2\sqrt{k}} = \frac{1}{2}\alpha\sqrt{\frac{2\pi}{k}} = \frac{1}{2}\alpha\sqrt{\lambda} = \frac{1}{2}v$

$\Rightarrow \boxed{v = 2v_g}$

(b) $\frac{i(px - Et)}{\hbar} = i(kx - \omega t)$ (for normal wave function).

$\Rightarrow \frac{p}{\hbar} = k, \quad \frac{E}{\hbar} = \omega = \frac{p^2}{2m\hbar} = \frac{\hbar k^2}{2m}$

$\Rightarrow v = \frac{\omega}{k} = \frac{E}{p} = \frac{p}{2m} = \frac{\hbar k}{2m}.$

$v_g = \frac{d\omega}{dk} = \frac{\hbar}{2m}2k = \frac{\hbar k}{m} = \frac{p}{m} \rightarrow$ corresponds to classical speed of particle.