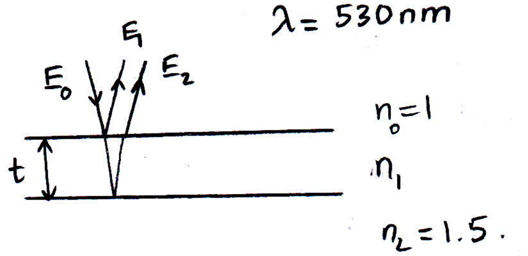
110B - Homework 6

$\lambda = 530\,nm$

①



To minimize reflection, we want rays 1 & 2 to have equal amplitude and a phase difference of π.

$$\frac{E_1}{E_0} = r_{01} = \frac{1-n_1}{1+n_1}, \quad \frac{E_2}{E_0} = t_{01} r_{12} t'_{10} = r_{12}(1-r_{01}^2) = \frac{n_1 - n_2}{n_1 + n_2}\left(1 - \left(\frac{1-n_1}{1+n_1}\right)^2\right)$$

$$\frac{1-n_1}{1+n_1} = \frac{n_1 - n_2}{n_1 + n_2}\left(1 - \left(\frac{1-n_1}{1+n_1}\right)^2\right)$$

$$\Rightarrow \frac{1-n_1}{1+n_1} = \frac{2n_1 - 3}{2n_1 + 3} - \left(\frac{2n_1 - 3}{2n_1 + 3}\right)\left(\frac{1-n_1}{1+n_1}\right)^2 \Rightarrow n_1 \approx 1.22$$ (the only $n_1 > 0$ solution).

$2nt = (m + \frac{1}{2})\lambda$ for destructive interference $(m \in \mathbb{N})$.

$\Rightarrow t = \frac{1}{2}\lambda \frac{1}{2n} = 108\ (nm)$

② To determine the refractive index: shine monochromatic light at the wedge at a fixed angle of θ and measure the intensity of the reflected beam:

$$\frac{I_R}{I_0} = R = \left(\frac{\alpha - \beta}{\alpha + \beta}\right)^2 \quad \text{where} \quad \alpha = \frac{1}{\cos\theta}\sqrt{1 - \frac{1}{n^2}\sin^2\theta} \quad \text{and } \beta = n.$$

We can measure I_0, I_R & θ $\Rightarrow$ we can find n.

To find the angle of the wedge: shine light onto the wedge and measure the separation between fringes:

$2tn = (m + \frac{1}{2})\lambda$ where t is the thickness of the wedge at the mth fringe.

Fringe separation: $\Delta x = \frac{\Delta t}{\sin\alpha} = \frac{\lambda}{2n\sin\alpha}$ where α is the angle of the wedge.
(between the m^{th} & $(m+1)^{th}$ fringes)

Note: to determine n, notice the reflection from both surfaces & their interferences. If the interference max & min are averaged, one get roughly the sum of I_R. If measure only the max, one can have the field add up.

(3)

air		$n=1$
SiO_2		$n=1.5$
Si		$n=3.2$

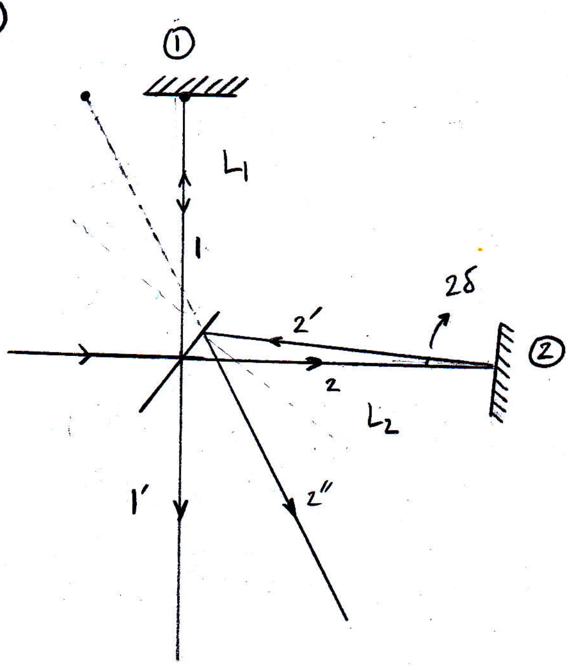
Both reflected rays are phase-shifted at the interfaces, so they are in phase with each other.

for constructive interference: $2nt = m\lambda$. $(n = 1.5)$.

t	310	345	390
λ	465	517.5	585
color	blue	green	yellow

(choose $m=2$ for visible range).

(4)



Assume mirror ② is misaligned by an angle δ from the vertical.

$L_1 = L_2 = L$.

The angle between rays 2 & 2' is 2δ. To a good approximation, the angle between rays 1' & 2'' is 2δ.

What we have is a two-point interference with $d = 2\delta L$.

The interference pattern is given by:

$$m\lambda = d\sin\theta = 2\delta L\sin\theta.$$

If one mirror is moved, i.e. $L_1 = L$, $L_2 = L + \Delta L$, then the path difference is $\sqrt{(2\delta L)^2 + (2\Delta L)^2} = 2\sqrt{\delta^2 L^2 + \Delta L^2}$

The diffraction pattern is: $m\lambda = 2\sqrt{\delta^2 L^2 + \Delta L^2}\,\sin\theta$.

$\Rightarrow$ As either ΔL or δ increases, the fringe separation decreases.

⑤ $I(x) = 2\int I_0\, dk + 2\int I_0 \cos 2kx\, dk$

for a range of wavelength from λ_1 to λ_2 : $I_0(\lambda) = 1$:

$$I(x) = 2\int_{2\pi/\lambda_2}^{2\pi/\lambda_1} dk + 2\int_{2\pi/\lambda_2}^{2\pi/\lambda_1} \cos 2kx\, dk$$

$$= 2(2\pi)\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) + 2\left.\frac{\sin(2kx)}{2x}\right|_{2\pi/\lambda_2}^{2\pi/\lambda_1}$$

$$= 4\pi\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) + \frac{1}{x}\left(\sin\frac{4\pi x}{\lambda_1} - \sin\frac{4\pi x}{\lambda_2}\right)$$

Plot this function for different $[\lambda_1, \lambda_2]$ and observe that the intensity profile in part (c) is the sum of those in parts (a) & (b).