PHYS 110 - HW7

① $\lambda = 1152.2\,nm$, $\theta = 6.2°$, $m = 10$.

$d\sin\theta = m\lambda$ for destructive interference

$$\Rightarrow d = \frac{10(1152.2\,nm)}{\sin(6.2°)} = 1.067 \times 10^5\,nm = 0.1067\,mm.$$

put in water: $n = 1.33$

$$\Rightarrow \theta = \arcsin\left(\frac{m\lambda}{nd}\right) = 4.657°.$$

② $D = 508\,cm$, $\lambda = 550\,nm$.

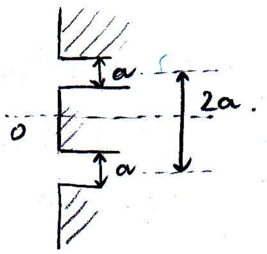
Limit of angular resolution: $\theta = \frac{1.22\lambda}{D} = 1.32 \times 10^{-7}$ rad.

$\Delta x = R\theta$, $R = 376\,000\,km = 3.76 \times 10^8\,m$

with telescope : $\Delta x_t = 49.63\,m$.

with eye : $\Delta x_e = R\left(\frac{1.22\lambda}{4mm}\right) = 6.3074 \times 10^4\,m \approx 63\,km$.

③ $$E \propto \int_{a/2}^{3a/2} e^{iks\sin\theta s}\,ds + \int_{-3a/2}^{-a/2} e^{iks\sin\theta}\,ds.$$



$$\int_{a/2}^{3a/2} e^{iks\sin\theta}\,ds = \frac{2e^{iak\sin\theta}}{k\sin\theta}\sin\left(\frac{ak\sin\theta}{2}\right)$$

$$= \frac{ae^{2i\alpha}}{\alpha}\sin\alpha \qquad \left(\alpha = \frac{ak\sin\theta}{2}\right).$$

$$\int_{-3a/2}^{-a/2} e^{iks\sin\theta}\,ds = \frac{ae^{-2i\alpha}}{\alpha}\sin\alpha$$

$$\rightarrow E \propto \frac{a}{\alpha}\sin\alpha\left(e^{2i\alpha} + e^{-2i\alpha}\right) = \frac{a}{\alpha}\sin\alpha\,(2\cos(2\alpha)).$$

$$\rightarrow I \propto 4\frac{a^2}{\alpha^2}\sin^2\alpha\cos^2(2\alpha) = 4\frac{a^2}{\frac{a^2k^2\sin^2\theta}{4}}\sin^2\left(\frac{a}{2}k\sin\theta\right)\cos^2(ak\sin\theta)$$

$$= \frac{1}{k^2\sin^2\theta}\sin^2\left(\frac{a}{2}k\sin\theta\right)\cos^2(ak\sin\theta).$$

④ $N = 1000\ mm^{-1}$, $S = 5cm$

$\lambda = 632nm$, $d_{laser} = 1cm$, $\theta_{in} = 30°$

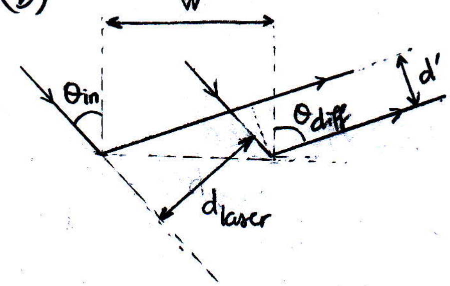
(a) $a(\sin\theta_{diff} + \sin\theta_{in}) = m\lambda \qquad (a = 1/N)$

$\Rightarrow \theta_{diff} = \arcsin(m\lambda N - \sin\theta_{in})$ and $|\sin\theta_{diff}| \leq 1.$

$\Rightarrow \quad -0.79 \leq m \leq 2.37$

$\Rightarrow \quad m \in \{0, 1, 2\} \Rightarrow \quad \theta_{diff} \in \{-30°, 7.585°, 49.82°\}$: 3 diffraction peaks

(b)



$\frac{d'}{w} = \sin(90° - \theta_{diff}) = \cos\theta_{diff}.$

$\frac{d_{laser}}{w} = \sin(90° - \theta_{in}) = \cos\theta_{in}$

$\Rightarrow d' = d_{laser}\frac{\cos\theta_{diff}}{\cos\theta_{in}}$

$\theta_{div} = \frac{\lambda}{\pi d'} = \frac{\lambda}{\pi d_{laser}}\frac{\cos\theta_{in}}{\cos\theta_{diff}} \quad \Rightarrow \theta_{div} \in \{2.01\times10^{-5}, 1.76\times10^{-5}, 2.7\times10^{-5}\}$ (rad).

c. $a(\sin\theta_{diff} + \sin\theta_{in}) = m\lambda.$

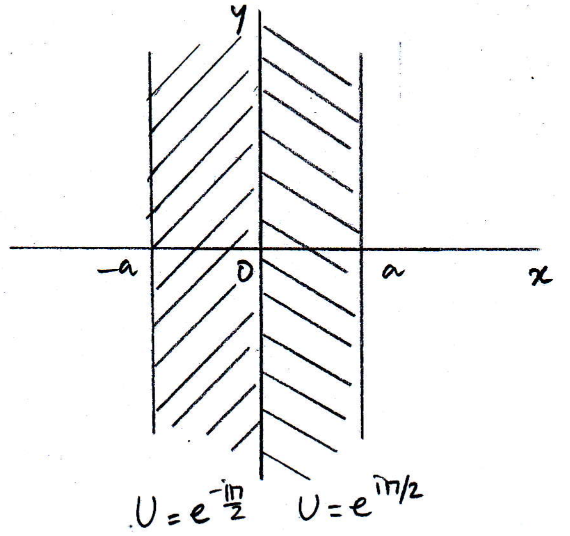
$\Rightarrow a\cos\theta_{diff}\,\delta\theta_{diff} = m\delta\lambda.$

We want $\delta\theta_{diff} \gg \theta_{div} \Leftrightarrow \frac{m\delta\lambda}{a\cos\theta_{diff}} \geq \frac{\lambda}{\pi d_{laser}}\frac{\cos\theta_{in}}{\cos\theta_{diff}}$

$\Rightarrow \delta\lambda \geq \frac{\lambda a}{\pi d_{laser}}\cos\theta_{in} \quad (m=1).$

$\Rightarrow \delta\lambda \geq 0.0174\ nm$

⑤ (a)



(b) Use a thin film to cover half of the area bounded by $x = a$ and $x = -a$ to introduce the π phase delay.

⑤ (c)

$$E \propto \iint U(x', y')\, e^{-i(k_x x' + k_y y')}\, dx'\, dy'$$

$$= \int_{-\infty}^{\infty} e^{-ik_y y'}\, dy' \int_{-\infty}^{\infty} U(x')\, e^{-ikx'}\, dx'$$

$$= 2\pi\, \delta(k_y) \left(\int_0^a e^{i\pi/2} e^{-ik_x x'}\, dx' + \int_{-a}^{0} e^{-i\pi/2} e^{-ik_x x'}\, dx' \right)$$

$$= 2\pi\, \delta(k_y) \left(\frac{1 - e^{-iak_x}}{k_x} + \frac{1 - e^{iak_x}}{k_x} \right)$$

$$= \frac{4\pi}{k_x}\, \delta(k_y) \left(1 - \cos(ak_x) \right)$$

$$I = \frac{1}{2} c \varepsilon_0 E_0^2 \Rightarrow I = I_0 \left(\frac{4\pi}{k_x} \right)^2 \delta(k_y)^2 \left(1 - \cos(ak_x) \right)^2$$

$$= I_0 \left(\frac{4\pi}{k_x} \right)^2 \delta(k_y)\, 4 \sin^4\left(\frac{ak_x}{2} \right)$$