PHYS 110B –

Homework 8 Solution

① Use Jones matrix :

Half-wave retarders: $M = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$ (Note: choice of Jones matrix is not unique).

(with FA at θ wrt. x-axis)

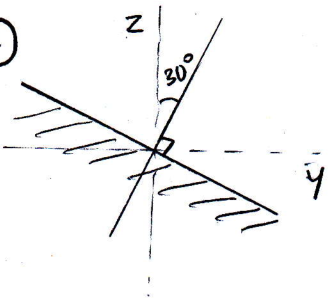
$$\Rightarrow M_1 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad M_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Rightarrow M_{21} = M_2 M_1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

(i) LP. at 0° : $V = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ (wrt. y-axis) $\longrightarrow M_{21}V = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$: LP at 90°.

(ii) LP at 45°: $V = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \rightarrow M_{21}V = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$: LP at −45°.

(iii) RCP : $V = \begin{pmatrix} 1 \\ -i \end{pmatrix} \rightarrow M_{21}V = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ -i \end{pmatrix} = \begin{pmatrix} i \\ 1 \end{pmatrix}$:

multiplied by $-i \rightarrow \begin{pmatrix} 1 \\ -i \end{pmatrix}$: RCP.

② 

(i) For x-polarized light: E_x is vibrating along x-axis

$\rightarrow n_{eff,x} = n_x = 1.4$.

For y-polarized light:

$k_y = \cos 30°\, k_0 = \frac{\sqrt{3}}{2} n_{eff} k_0$; $k_z = \sin 30°\, k_0 = \frac{1}{2} n_{eff} k_0$

$$k_0^2 = \frac{k_y^2}{n_z^2} + \frac{k_z^2}{n_y^2} = \frac{3}{4} n_{eff}^2 \frac{k_0^2}{n_z^2} + \frac{1}{4} n_{eff}^2 \frac{k_0^2}{n_y^2} \Rightarrow 1 = n_{eff}^2 \left(\frac{3}{4n_z^2} + \frac{1}{4n_y^2} \right).$$

$$\Rightarrow n_{eff,y} = \left(\frac{3}{4(1.41)^2} + \frac{1}{4(1.4)^2} \right)^{-1/2} = 1.4075.$$

(ii) QWP: $\Delta\varphi = \pi/2 = k\underset{\substack{\uparrow \\ \text{path difference}}}{\Delta L} = k(n_{eff,y} - n_{eff,x})d = \frac{2\pi}{\lambda}(n_{eff,y} - n_{eff,x})d.$

$$\Rightarrow d = \frac{\lambda}{4}(1.4075 - 1.4)^{-1} = 2.7 \times 10^{-5}\ \text{m} = 27\,\mu\text{m}.$$

③ (i) $\Delta\varphi = \pi = k\Delta L = k_o d\left(n_{eff} - n(E_{bias}=0)\right) = \frac{2\pi}{\lambda} d\,(3 + 10^{-12} E_{bias} - 3)$

$\Rightarrow E_{bias} = \frac{\lambda}{2d}(10^{12})$ $\quad$ ($d = 10\,cm$, $t = 1\,mm$).

$\Rightarrow V_{bias} = Et = \frac{\lambda t}{2d}(10^{12}) = \frac{\lambda}{2}(10^{10})$.

(ii) $I = \frac{1}{2}I_o + \frac{1}{2}I_o \cos\Delta\varphi = \frac{I_o}{2}(1 + \cos\Delta\varphi)$.

$= \frac{I_o}{2}\left(1 + \cos\left[k_o d(10^{-12}E_{bias})\right]\right) = \frac{I_o}{2}\left(1 + \cos\left[k_o d\,(10^{-12})\frac{V_{bias}}{t}\right]\right)$

$= \frac{I_o}{2}\left(1 + \cos(10^{-10} k_o V_{bias})\right)$.

(iii) $I = 0.5 I_o \Rightarrow \cos(10^{-10} k_o V_{bias}) = 0 \Rightarrow 10^{-10}\frac{2\pi}{\lambda}V_{bias} = \frac{\pi}{2} \Rightarrow V_{bias} = \frac{\lambda}{4}10$

④ $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1}{1.5}\right) = 0.73$ rad. $\approx 42°$.

$\theta_i = 1.2\,\theta_c \simeq 0.88$

$n_1 \sin\theta_i = n_2 \sin\theta_t \Rightarrow \sin\theta_t = 1.15$

$\Rightarrow \alpha = \frac{\cos\theta_t}{\cos\theta_i} = \frac{\sqrt{1-\sin^2\theta_t}}{\cos\theta_i} = \frac{i\sqrt{\sin^2\theta_t - 1}}{\cos\theta_t} \simeq 0.89i$

$\beta = \frac{\mu_1 n_2}{\mu_2 n_1} \approx \frac{n_2}{n_1} = \frac{2}{3}$.

p-polarized light: $Ae^{i\Delta\varphi} = r = \frac{\alpha-\beta}{\alpha+\beta} = 0.28 + .96i = Ae^{i\arctan(.96/.28)}$

$\Rightarrow \Delta\varphi = 1.28$ rad.

s-polarized light: $Ae^{i\Delta\varphi} = r = \frac{1-\alpha\beta}{1+\alpha\beta} = 0.48 - 0.88i \Rightarrow \Delta\varphi = \tan^{-1}\left(\frac{-.88}{.48}\right)$

$\Rightarrow \Delta\varphi = -1.07$ rad.

5 (i) $kd\cos\theta = m\pi \Rightarrow k\cos\theta = \frac{m\pi}{d} = k_\perp$.

$$k_{//} = k\sin\theta = k\sqrt{1-\cos^2\theta} = \sqrt{k^2 - \left(\frac{m\pi}{d}\right)^2} = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{d}\right)^2}$$

Condition: $\frac{\omega}{c} \geq \frac{m\pi}{d} \Rightarrow m \leq \frac{\omega d}{\pi c} = \frac{2\nu d}{c} = \frac{2(100\,\text{GHz})(5\,\text{cm})}{c} = 33.3..$

$\Rightarrow$ 33 guided TE modes exist in this waveguide.

(ii) $$v_g = c\sin\theta = c\frac{k_{//}}{k} = \frac{c}{k}\sqrt{k^2 - \left(\frac{m\pi}{d}\right)^2} = c\sqrt{1-\left(\frac{m\pi}{kd}\right)^2} = c\sqrt{1-\left(\frac{mc}{2\nu d}\right)^2}.$$

$k = \frac{2\pi\nu}{c}$ $\qquad (m = 1, 2, \ldots, 33)$.

(iii) Fastest guided TE mode: $m = 1 \Rightarrow \beta d = m\pi = \pi. \Rightarrow \beta y = \frac{\pi}{d}y$.

$$E_x(y) = E_0 \sin\left(\frac{\pi y}{d}\right) \Rightarrow \tilde{E} = \tilde{E}_0 \sin\left(\frac{\pi y}{d}\right) e^{i(kz-\omega t)}\hat{x} = \tilde{E}_x \hat{x}.$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla\times\tilde{E} = \partial_z(\tilde{E}_x)\hat{y} - \partial_y(\tilde{E}_x)\hat{z}$$
$$= ik\tilde{E}_0 \sin\left(\frac{\pi y}{d}\right) e^{i(kz-\omega t)}\hat{y} - \frac{\pi}{d}\tilde{E}_0 \cos\left(\frac{\pi y}{d}\right) e^{i(kz-\omega t)}\hat{z}$$
$$= \tilde{E}_0 e^{i(kz-\omega t)}\left(ik\sin\left(\frac{\pi y}{d}\right)\hat{y} - \frac{\pi}{d}\cos\left(\frac{\pi y}{d}\right)\hat{z}\right)$$

$$\Rightarrow \vec{B} = \frac{i}{\omega}\tilde{E}_0 e^{i(kz-\omega t)}\left(ik\sin\left(\frac{\pi y}{d}\right)\hat{y} - \frac{\pi}{d}\cos\left(\frac{\pi y}{d}\right)\hat{z}\right)$$
$$= \left[-\frac{1}{c}\sin\left(\frac{\pi y}{d}\right)\hat{y} - \frac{i\pi}{\omega d}\cos\left(\frac{\pi y}{d}\right)\hat{z}\right]\tilde{E}_0 e^{i(kz-\omega t)}$$