① $p^\mu = \begin{pmatrix} E \\ E\cos\theta \\ E\sin\theta \\ 0 \end{pmatrix}$, $E = h\nu$

(a) $E' = E\gamma - E\cos\theta\, v\gamma = h\nu\gamma(1 - v\cos\theta) = h\nu'$

$\Rightarrow \boxed{\nu' = \gamma\nu(1 - v\cos\theta)}$.

(b) No Doppler shift $\equiv \gamma(1 - v\cos\theta) = 1 \rightarrow \boxed{\cos\theta = \frac{1}{v}\left(1 - \sqrt{1 - v^2}\right)}$

(c) Suppose the proton is travelling in the $+x$-direction with velocity v in the rest frame, where the photon has energy E_γ^i.

In the proton's frame: the photon's energy is $E_\gamma^{i'}$, the angle between the photon's momentum & x-axis is θ'.

After scattering, the photon travels at $\theta' - \varphi'$, where φ' is the angle between the initial & final photons.

$$E_\gamma^i = E_\gamma^{i'}\gamma(1 + v\cos\theta')$$

$$E_\gamma^{f'} = \frac{E_\gamma^{i'} m_p}{m_p + E_\gamma^{i'}(1 - \cos\varphi')} \leftarrow \left(c = 1,\ \lambda^{f'} - \lambda^{i'} = \frac{h}{m_p c}(1 - \cos\theta)\right).$$

↑ scattering angle

$$E_\gamma^f = E_\gamma^{f'}\gamma\left(1 + v\cos(\theta' - \varphi')\right) = \frac{E_\gamma^{i'} m_p}{\left(m_p + E_\gamma^{i'}(1 - \cos\varphi')\right)}\gamma\left(1 + v\cos(\theta' - \varphi')\right)$$

$$= \frac{E_\gamma^i m_p}{m_p + E_\gamma^i \frac{1 - \cos\varphi'}{\gamma(1 + v\cos\theta')}} \frac{1 + v\cos(\theta' - \varphi')}{1 + v\cos\theta'}$$

② Griffiths' (10.65): E of a moving point charge

$$\vec{E}(\vec{r},t) = \frac{q}{4\pi\varepsilon_0}\frac{\imath}{(\vec{\imath}\cdot\vec{u})^3}\left[(c^2-v^2)\vec{u} + \vec{\imath}\times(\vec{u}\times\vec{a})\right] \qquad (\vec{u} \equiv c\hat{\imath} - \vec{v})$$

The first term falls off as $\frac{1}{\imath^2}$, if $\vec{v}$ and $\vec{a}$ are both 0, then:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{\imath^2}\hat{\imath}$$: velocity field or generalized Coulomb field.

The second term falls off as $\frac{1}{\imath}$ ⇒ dominates at large distance

→ radiation field or acceleration field.

If the charge is instantaneously at rest at t_r, then $\vec{u} = c\hat{\imath}$:

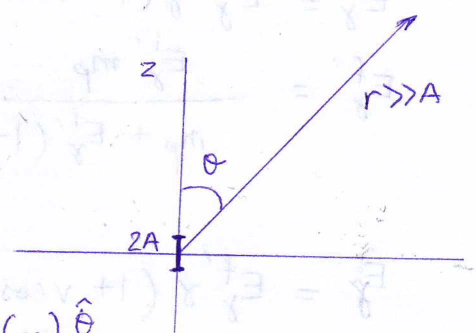
$$\vec{E}_{rad} = \frac{q}{4\pi\varepsilon_0}\frac{\imath}{(\vec{\imath}\cdot c\hat{\imath})^3}\left[\vec{\imath}\times(c\hat{\imath}\times\vec{a})\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{\imath}{\imath^3 c^3}\left[c\hat{\imath}(\vec{\imath}\cdot\vec{a}) - \vec{a}(\vec{\imath}\cdot c\hat{\imath})\right] \qquad \left(A\times(B\times C) = B(A\cdot C) - C(A\cdot B)\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{1}{\imath^2 c^2}\left(\hat{\imath}(\imath a_{\parallel}) - \imath a_{\parallel}\hat{\imath} - \imath\vec{a}_{\perp}\right) \qquad (a_{\parallel} \parallel \hat{\imath},\ a_{\perp}\perp\hat{\imath})$$

$$\Rightarrow \boxed{\vec{E}_{rad} = \frac{-q}{4\pi\varepsilon_0}\frac{1}{\imath c^2}\vec{a}_{\perp}}$$

$$\vec{a}_{\perp} = \vec{a}_{\perp}(t - \tfrac{\imath}{c}) = -a(t - \tfrac{\imath}{c})\sin\theta\,\hat{\theta}$$



$$z = A\cos\omega t \Rightarrow a = \ddot{z} = -\omega^2 A\cos\omega t$$

$$\Rightarrow \boxed{\vec{E} = -\frac{q}{4\pi\varepsilon_0}\frac{1}{\imath c^2}\omega^2 A\cos(\omega(t-\tfrac{\imath}{c}))\sin\theta\,\hat{\theta}} = (\ldots)\hat{\theta}$$

Griffiths (10.66):

$$\vec{B} = \frac{1}{c}\hat{\imath}\times\vec{E} = \frac{1}{c}(\ldots)\hat{\imath}\times\hat{\theta} = \boxed{-\frac{q}{4\pi\varepsilon_0\imath c^3}\omega^2 A\cos(\omega(t-\tfrac{\imath}{c}))\sin\theta\,\hat{\phi}}$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{1}{\mu_0 c}(\ldots)^2\,\hat{\theta}\times(\hat{\imath}\times\hat{\theta}) = \frac{1}{\mu_0 c}(\ldots)^2\hat{\imath}.$$

$$P = \int\vec{S}\cdot d\vec{A} = \int_{-\pi/2}^{\pi/2}\frac{1}{\mu_0 c}(\ldots)^2\hat{\imath}\ 4\pi\imath^2\sin\theta\, d\theta\,\hat{\imath} = (\ldots)\int_{-\pi/2}^{\pi/2}\sin^3\theta\, d\theta$$

(integrate over a spherical surface of radius $\imath$).

$$\langle P\rangle = q^2\frac{\omega^4 A^2}{12\pi c^3\varepsilon_0}$$: rate of energy loss, same as what we would get from Larmor's formula.

(3) (a) $L' = \frac{L}{\gamma} = 20m(1-0.8^2)^{1/2} = 12m$

(b) $\Delta t = \frac{\Delta x}{v} = \frac{15m - 12m}{0.8c} = 1.25 \times 10^{-8} s.$

$\Delta s^2 = -c^2 \Delta t^2 + \Delta x^2 = -\left(\frac{15m}{4}\right)^2 + (15m)^2 = 211 m^2$: spacelike.

(c) Length of pole: 20m.

Length of barn: $L_b' = 15m(1-0.8^2)^{1/2} = 9m.$

(d) Spacelike separated events have no unique time ordering, so the runner does not believe that the pole is entirely in the barn when its front end hits the barn.