PHYS 110 - Midterm 1

Solution.

① $\chi = 3 - 0.01\lambda + 0.0001i$, $\lambda = 0.5\,\mu m$

(a) $\varepsilon = 1+\chi = \varepsilon' + i\varepsilon'' = 4 - 0.01\lambda + 0.0001i$

$n = \sqrt{\varepsilon} = n' + in'' = 1.99875 - 0.000025i$

$$\text{Skin depth} = \frac{1}{2n''k_0} = \frac{1}{2n''\left(\frac{2\pi}{\lambda}\right)} = \frac{\lambda}{4\pi n''} = \frac{0.5\,\mu m}{4\pi(0.000025)} \approx \boxed{1.59 \times 10^{-3}\,(m)}$$

(b) $v_p = \frac{c}{n} \approx \frac{c}{n'}$ because $n'' \ll n'$

$\Rightarrow \boxed{v_p \approx 0.5c}$

$k = nk_0 = n'k_0 + in''k_0 = (n' + in'')\frac{\omega}{c} = \frac{n}{c}\omega.$

$\Rightarrow v_g = \frac{1}{\partial k/\partial\omega} = \frac{1}{n/c} = \frac{c}{n} = v_p.$

② $P = P_0 \frac{\sin^2\theta}{r^2}$, $P_0 = A\omega_0^4$ (A is a constant).

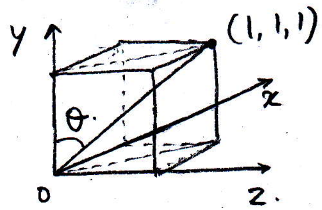
(a) Incident light is polarized in the y-direction $\Rightarrow \vec{p}$ is in the y-direction.

Detector at $(0,1,0)$: $\theta = 0 \Rightarrow P = 0.$

(b) Detector at $(1,1,0)$: $\theta = \frac{\pi}{4}$, $r = \sqrt{1+1} = \sqrt{2} \Rightarrow P = \left(\frac{\sqrt{2}}{2}\right)^2 \frac{1}{2} P_0 = \frac{P_0}{4}$

(c) Detector at $(1,0,1)$: $\theta = \frac{\pi}{2}$, $r = \sqrt{2} \Rightarrow P = \frac{1}{2}P_0.$

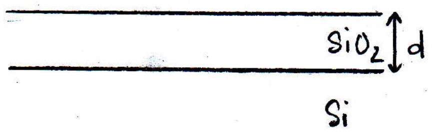
(d) Detector at $(1,1,1)$: $r = \sqrt{3}$, $\sin\theta = \frac{\sqrt{2}}{\sqrt{3}}.$



$\Rightarrow P = P_0 \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9} P_0.$

(e) $\lambda = \frac{1}{2}\lambda_0 \Rightarrow \omega = 2\omega_0 \Rightarrow P = 16P_0.$

③



$n_1 = 1.5$ $\qquad$ $P = 1W.$

$n_2 = 3 + 0.01i$ at $\lambda = 800nm$.

(1) Reflection at top surface: $r_1 = \dfrac{1-n_1}{1+n_1} = -0.2$

Reflection at bottom surface: $r_2 = \dfrac{n_1 - n_2}{n_1 + n_2} \approx \dfrac{1.5-3}{1.5+3} \simeq -0.33$ (approximate $n_2 = n_2'$, $n_2' \gg n_2''$)

$E_1 = r_1 E_0 = -0.2 E_0$

$E_2 = r_2 t t' E_0 = r_2 (1 - r_1^2) E_0 \approx -0.32 E_0$

$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\Delta\phi_2)$ ($\Delta\phi = \frac{2\pi}{\lambda}(2dn)$, assuming normal incidence)

$= E_1^2 + E_2^2 + 2 E_1 E_2 \cos(2\pi dn/\lambda)$

$I \approx 0.14 E_0^2 + 0.13 E_0^2 \cos(2\pi dn/\lambda)$.

$\Rightarrow I_{max} = 0.27 E_0^2$

$\cos \frac{2\pi}{\lambda} dn = 0 \Leftrightarrow n \frac{2\pi}{\lambda} d = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots \Leftrightarrow nd = \frac{\lambda}{4}, \frac{3\lambda}{4}, \ldots$

$\cos \frac{2\pi}{\lambda} dn = 1 \Leftrightarrow n \frac{2\pi d}{\lambda} = 0, \pi, 2\pi, \ldots \Leftrightarrow nd = 0, \frac{\lambda}{2}, \lambda, \ldots$

(Alternatively: $m\lambda = 2n_1 d$ for maxima $\Rightarrow n_1 d = 0, \frac{\lambda}{2}, \ldots$)

$n_1 d = \frac{3}{2} d. \Rightarrow$ maxima at $d = 0, \frac{\lambda}{3}, \frac{2\lambda}{3}, \ldots = 0, \frac{800}{3} nm, \ldots$

minima at $d = \frac{\lambda}{6}, \frac{\lambda}{2}, \ldots = \frac{400}{3} nm, 400 nm, \ldots$

(2) $\text{Force} = (\text{radiation pressure}) \times (\text{area}) = \frac{(\text{intensity})}{c} \times (\text{area}) = \frac{(\text{power})}{(\text{area})} \frac{(\text{area})}{c}$

Because the momentum of the reflected light is in the opposite direction of the incident light, the change in power is the sum of the magnitudes of the power & the reflected power.

$$\Rightarrow \text{Force} = \frac{\Delta P}{c} = \frac{(1+0.27)W}{c} = 4.23 \times 10^{-9} N.$$

(3) To have zero reflection: $E_1 = E_2$ and $\Delta\phi = \pi$.

$\Delta\phi = \frac{2\pi n}{\lambda}(2d\cos\theta) = \pi \Rightarrow d\cos\theta = \frac{\lambda}{4n}$, where $\theta = \theta_2$ is the angle of the ray reflected off the 2nd interface.

$$E_1 = E_2 \Leftrightarrow r_1 = r_2(1-r_1^2)$$

$$r_{ab} = \frac{n_a\cos\theta_a - n_b\cos\theta_b}{n_a\cos\theta_a + n_b\cos\theta_b} \quad \text{for s-wave}$$

$$\Rightarrow \frac{\cos\theta_0 - 1.5\cos\theta_1}{\cos\theta_0 + 1.5\cos\theta_1} = \frac{1.5\cos\theta_1 - 3\cos\theta_2}{1.5\cos\theta_1 + 3\cos\theta_2}\left[1 - \left(\frac{\cos\theta_0 - 1.5\cos\theta_1}{\cos\theta_0 + 1.5\cos\theta_1}\right)^2\right]$$

$$\cos\theta_1 = \sqrt{1-\sin\theta_1^2} = \sqrt{1 - \frac{\sin^2\theta_0}{(1.5)^2}} = \sqrt{1 - \frac{4}{9}\sin^2\theta_0}$$

$$\cos\theta_2 = \sqrt{1 - \left(\frac{1.5\sin\theta_1}{3}\right)^2} = \sqrt{1 - \frac{1}{9}\sin^2\theta_0}$$

Solve for θ_0.