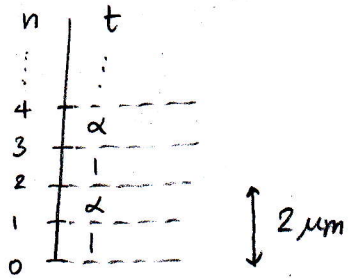
PHYS 110 - Midterm 2

Solutions.



① $t = 1$ for $2n < y < 2n+1$.
$t = \alpha$ for $2n+1 < y < 2n+2$, $n \in \mathbb{N}$

(i) $b(\sin\theta_{diff} - \sin\theta_i) = m\lambda$.

$\theta_i = 30°$, $\lambda = 800nm$, $b = 2\mu m$.

$\Rightarrow b\left(\sin\theta_{diff} - \frac{1}{2}\right) = m\lambda$.

$\Rightarrow \left|\sin\theta_{diff}\right| = \left|\frac{1}{2} + \frac{m\lambda}{b}\right| = \left|\frac{1}{2} + m\,\frac{0.8\mu m}{2\mu m}\right| = \left|\frac{1}{2} + 0.4m\right| \leq 1 \Rightarrow m = 1, 0, -1, -2, -3$

$\Rightarrow$ 5 diff. orders.

(ii) $m = -1$: $\theta_{diff} = -17.5°$

$D' = D\,\frac{\cos\theta_{diff}}{\cos\theta_i} = 0.01m \times \frac{\cos(-17.5°)}{\cos(30°)} = 0.011\ m$

$\theta_{div} = \frac{\lambda}{\pi D'} = \frac{800nm}{\pi(0.011\ m)} = 0.000023\ rad = 0.0013°$.

(iii) The diffraction for $t=1$ region & $t=\alpha$ region are out of phase by π

$\Rightarrow I_{diff} = (E_0 - \alpha E_0)^2 = (0.5E_0)^2 = 0.25E_0^2 = 0.25I_0$.

(2) (i) Place the quartz plate at B.

We want $I_0/2$ to be transmitted through the 2nd polarizer

$\Rightarrow$ the quartz plate should rotate y-polarized light into x-polarized light

$\rightarrow$ want half wave plate with slow axis at 45°.

$$\Delta n k_0 d = \pi \quad \Rightarrow d = \frac{\pi}{\Delta n k_0} = \frac{\pi}{|n_1 - n_2|} \frac{\lambda}{2\pi} = 4 \times 10^{-5}\ m = 40\ \mu m$$

(ii) Place a QWP at A so that the intensity is not halved when it passes through A

$$\Delta n k_0 d = \frac{\pi}{2} \quad \Rightarrow d = 20\ \mu m.$$

Orientation: 45° wrt the 1st polarizer so that the RCP light is changed to −45° LP light after passing through the QWP and is aligned with the 1st polarizer.

(3) (i) $nk_0 d\cos\theta = m\pi \Rightarrow nk_0\cos\theta = \frac{m\pi}{d} = k_\perp$; $k_0 = \frac{2\pi}{\lambda_0} = \frac{2\pi\nu}{c}$

$k_{//}^2 = (nk_0)^2 - (k_\perp)^2 \Rightarrow k_{//} = \sqrt{(nk_0)^2 - \left(\frac{m\pi}{d}\right)^2}$

$\Rightarrow nk_0 \geq \frac{m\pi}{d} \Rightarrow m \leq \frac{nk_0 d}{\pi} = \frac{(1.5)(2\times10^{-3}m)}{\pi}\frac{2\pi(100\times10^9 Hz)}{c} = 2.$

$\Rightarrow$ number of modes $= 2$.

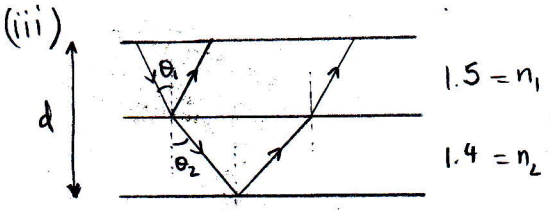
(ii) Lowest mode: $m=1$.

$v_p = \frac{\omega}{k_{//}} = \frac{2\pi\nu}{\sqrt{(nk_0)^2 - (\pi/d)^2}} = 2.31\times10^8 (m/s)$

$v_g = \frac{d\omega}{dk_{//}} = \left(\frac{dk_{//}}{d\omega}\right)^{-1} = \left[\frac{d}{d\omega}\left((n\frac{\omega}{c})^2 - \left(\frac{\pi}{d}\right)^2\right)^{1/2}\right]^{-1}$

$= \left[\left((n\frac{\omega}{c})^2 - \left(\frac{\pi}{d}\right)^2\right)^{-1/2}\frac{n^2}{c^2}\omega.\right]^{-1} = \frac{k_{//}c^2}{n^2\omega} = 1.73\times10^8 (m/s)$

(iii)



$n_1\sin\theta_1 = n_2\sin\theta_2$

$n_1 k_0\cos\theta_1 + n_2 k_0\cos\theta_2 = \frac{m\pi}{d/2}$

$k_0 = \frac{2\pi\nu}{c}$

$\Rightarrow n_1\cos\theta_1 + n_2\cos\theta_2 = \frac{2m\pi}{dk_0} = \frac{mc}{d\nu}$.

(iv) $\cos\theta_2 = \sqrt{1-\sin^2\theta_2} = \sqrt{1-\left(\frac{n_1}{n_2}\sin\theta_1\right)^2}$

$\Rightarrow n_1\sqrt{1-\sin^2\theta_1} + n_2\sqrt{1-\left(\frac{n_1}{n_2}\sin\theta_1\right)^2} = \frac{mc}{d\nu}$.

$\sin\theta_1 = x$, $A = \frac{mc}{d\nu}$

$n_2\left(1-\left(\frac{n_1}{n_2}x\right)^2\right)^{1/2} = -n_1\sqrt{1-x^2} + A$

$\Rightarrow n_2^2\left(1-\frac{n_1^2}{n_2^2}x^2\right) = A^2 + n_1^2(1-x^2) - 2An_1\sqrt{1-x^2}$.

$n_2^2 - n_1^2x^2 = A^2 + n_1^2 - n_1^2x^2 - 2An_1\sqrt{1-x^2}$

$n_2^2 - A^2 - n_1^2 = -2An_1\sqrt{1-x^2}$.

$\Rightarrow \sqrt{1-x^2} = \frac{A^2+n_1^2-n_2^2}{2An_1} \Rightarrow 1-x^2 = \left(\frac{A^2+n_1^2-n_2^2}{2An_1}\right)^2 \Rightarrow x = \sqrt{1-\frac{(A^2+n_1^2-n_2^2)^2}{4A_1^2n_1^2}}$

$$\sqrt{1-x^2} = \cos\theta_1 \leq 1 \Rightarrow \frac{(mc/d\nu)^2 + n_1^2 - n_2^2}{2\, mc/d\nu\, n_1} \leq 1.$$

$$\Rightarrow \left(\frac{mc}{d\nu}\right)^2 - 2\,\frac{mc}{d\nu} n_1 \leq n_2^2 - n_1^2.$$

$$\Rightarrow 2.25\, m^2 - 4.5\, m \leq 0.29$$

$$\Rightarrow m \leq 2.0625$$

$\Rightarrow$ 2 TE modes ($m = 1, 2$).

For $m = 1$:

$$k_z = n_1 k_0 \sin\theta_1 = n_1 \frac{2\pi\nu}{c} x_{m=1} \simeq 2593.3$$

$(k_z = n_2 k_0 \sin\theta_2)$.